Digital Curvature Evolution Model for Image Segmentation

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Discrete Geometry for Computery Imagery

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Presentation plan

Introduction

- Motivation problems
- Regularization in imaging
- Curvature as regularization
- Discretization and multigrid convergence

Contribution

- Curve Evolution Model
- Interpretation
- Discussion and application

Conclusion

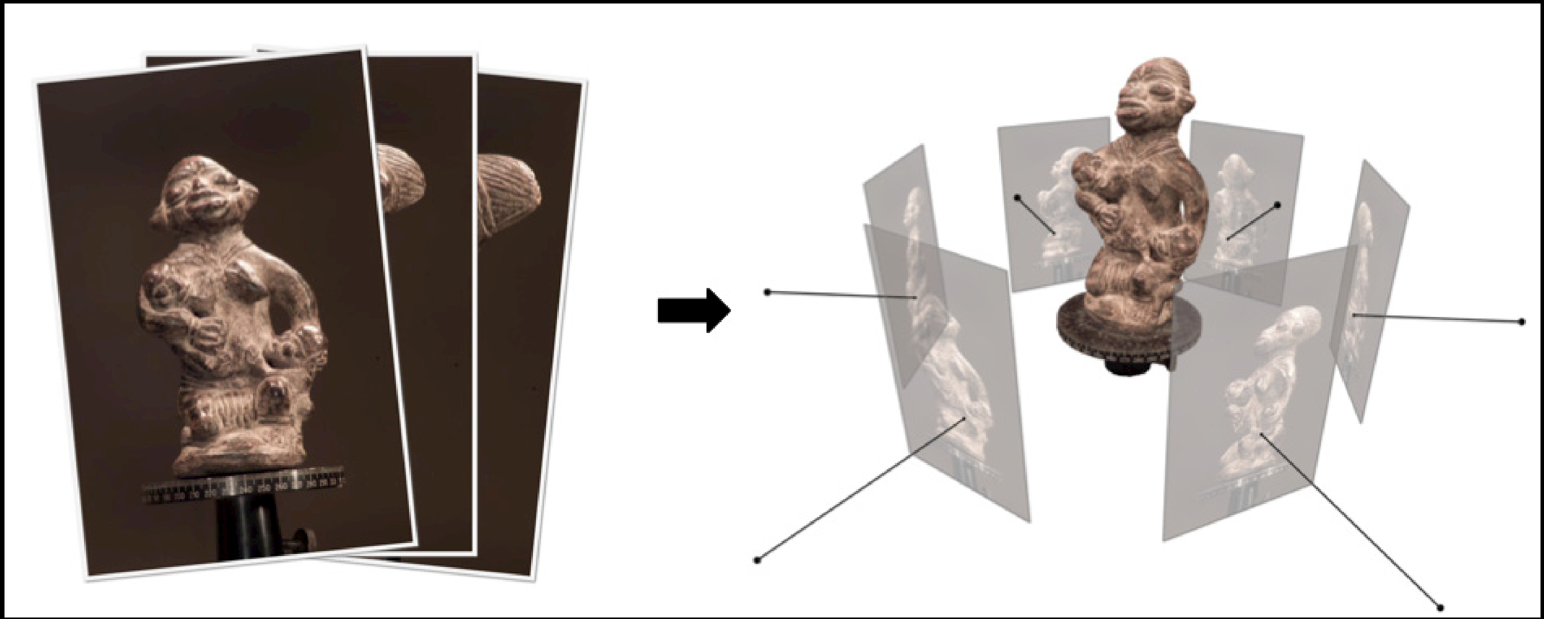
Image segmentation

Image segmentation

Denoising

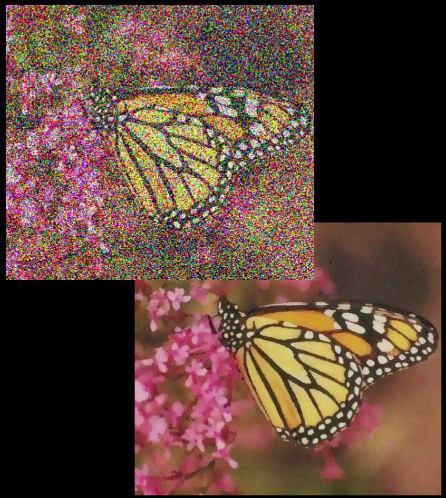
[Unger, Werlberger, 2011]

3D Reconstruction



[Furukawa, Hernández 2015]

Segmentation



Denoising

3D Reconstruction

Segmentation Denoising 3D Reconstruction

Solving Strategy

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Let Ω be the image space

$$u^\star = \arg\min_{u \in \Omega} E(u)$$

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Model for denoising

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Model for denoising

$$E(u) = \alpha ||g - u||^2 + \beta ||\nabla u||^2,$$

where g is the noisy (input) image.

Solving Strategy

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$$u^\star = \arg\min_{u \in \Omega} E(u)$$

Model for denoising

Solution resemble input image

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where g is the noisy (input) image.

Solving Strategy

Let Ω be the image space

$$u^{\star} = \arg\min_{u \in \Omega} E(u)$$

Model for denoising

Solution resemble input image

$$E(u) = \alpha ||g - u||^2 + \beta ||\nabla u||^2,$$

Solution should be smooth

where g is the noisy (input) image.

Image segmentation

Image segmentation

Mumford-Shah model

Image segmentation

Mumford-Shah model

$$E(u,K) = \int_{\Omega - K} (g-u)^2 dx + \lambda \int_{\Omega - K} ||\nabla u||^2 dx + \int_K dx$$

Image segmentation

Mumford-Shah model

$$E(u,K) = \int_{\Omega - K} (g-u)^2 dx + \lambda \int_{\Omega - K} ||\nabla u||^2 dx + \int_K dx$$

↑ Similar to original image

Image segmentation

Mumford-Shah model

$$E(u,K) = \int_{\Omega - K} (g-u)^2 dx + \lambda \int_{\Omega - K} ||\nabla u||^2 dx + \int_K dx$$

Piecewise smooth

Image segmentation

Mumford-Shah model

$$E(u,K) = \int_{\Omega - K} (g-u)^2 dx + \lambda \int_{\Omega - K} ||\nabla u||^2 dx + \int_K dx$$

Small perimeter

Image segmentation

Mumford-Shah model

$$E(u,K) = \int_{\Omega - K} (g-u)^2 dx + \lambda \int_{\Omega - K} ||\nabla u||^2 dx + \int_K dx$$

Binary piece-wise smooth **[Chan; Vese, 2001]**

Image segmentation

Mumford-Shah model

$$E(u,K) = \int_{\Omega - K} (g-u)^2 dx + \lambda \int_{\Omega - K} ||\nabla u||^2 dx + \int_K dx$$

Binary piece-wise smooth **[Chan; Vese, 2001]**

Optimization of Ambrosio Tortorelli energy **[Foare; Lachaud; Talbot, 2016]**

Curvature as regularization in segmentation

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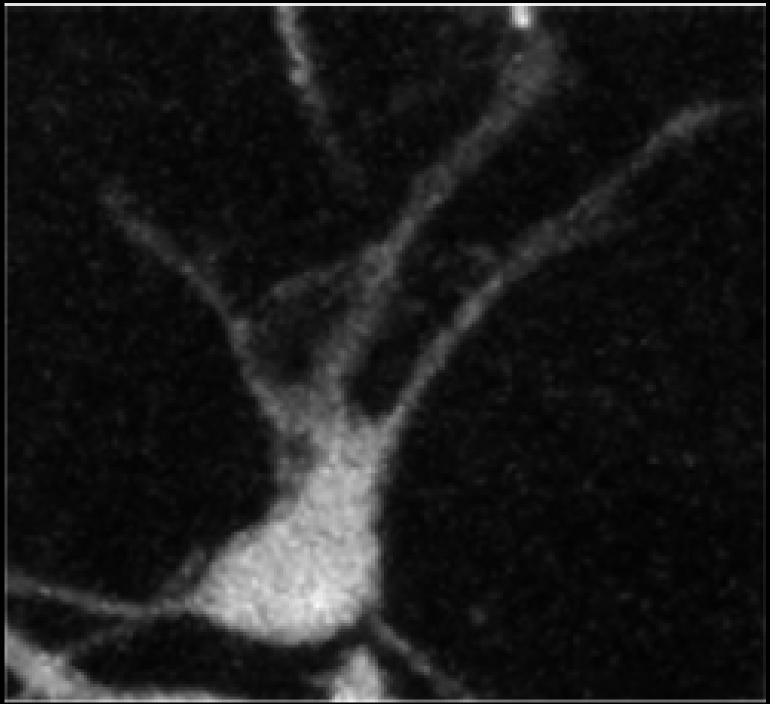
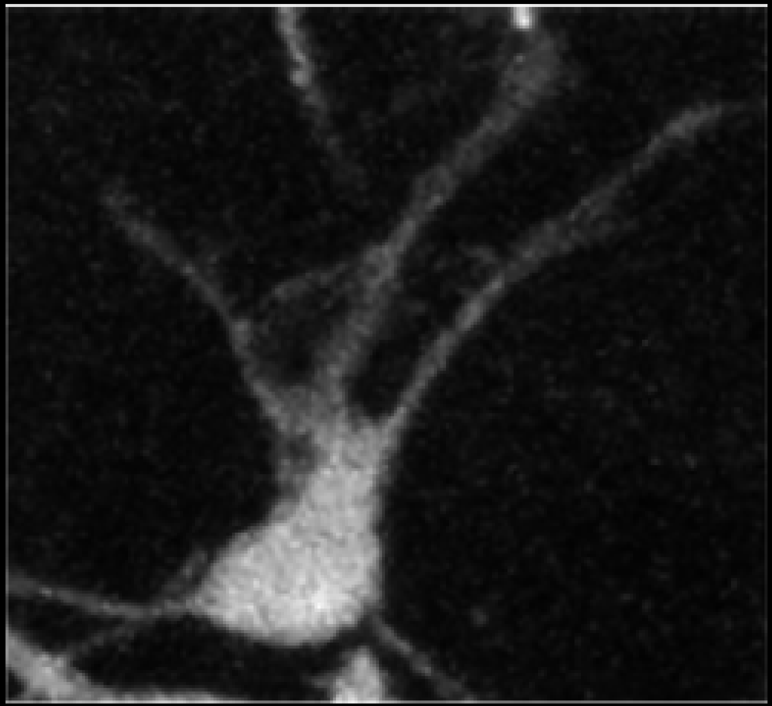


[El-Zehiry, 2010]

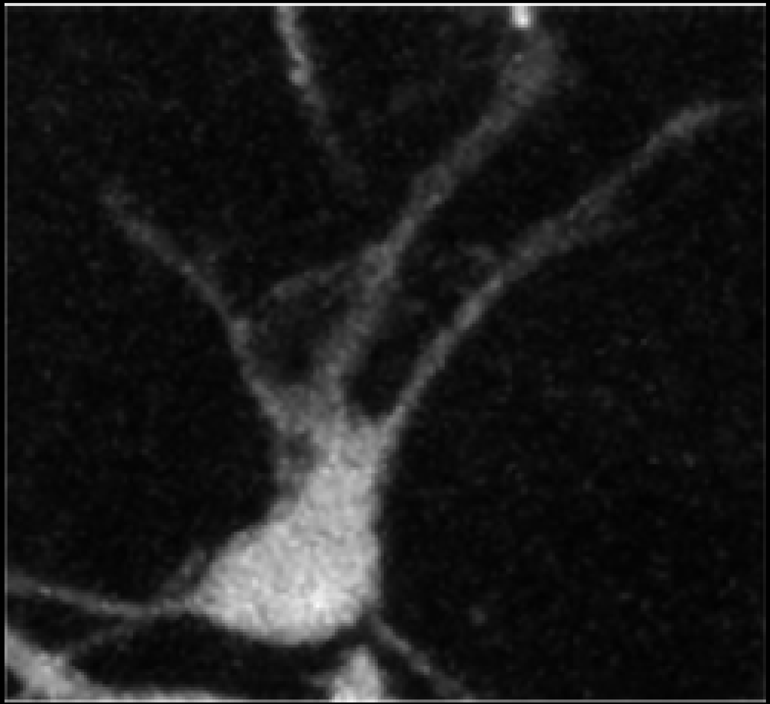
Curvature as regularization in segmentation



[El-Zehiry, 2010]

Data term

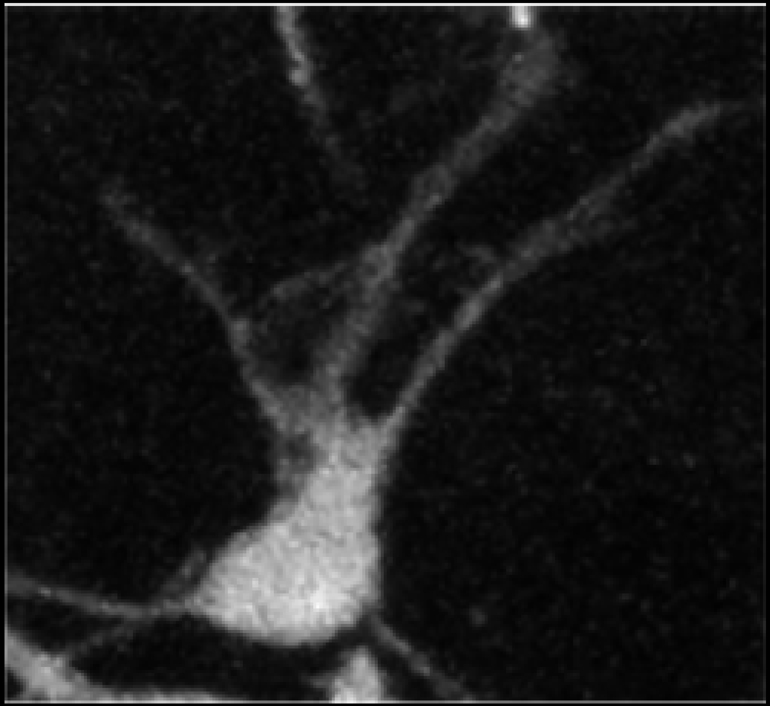
Curvature as regularization in segmentation



[El-Zehiry, 2010]

Data + Perimeter term

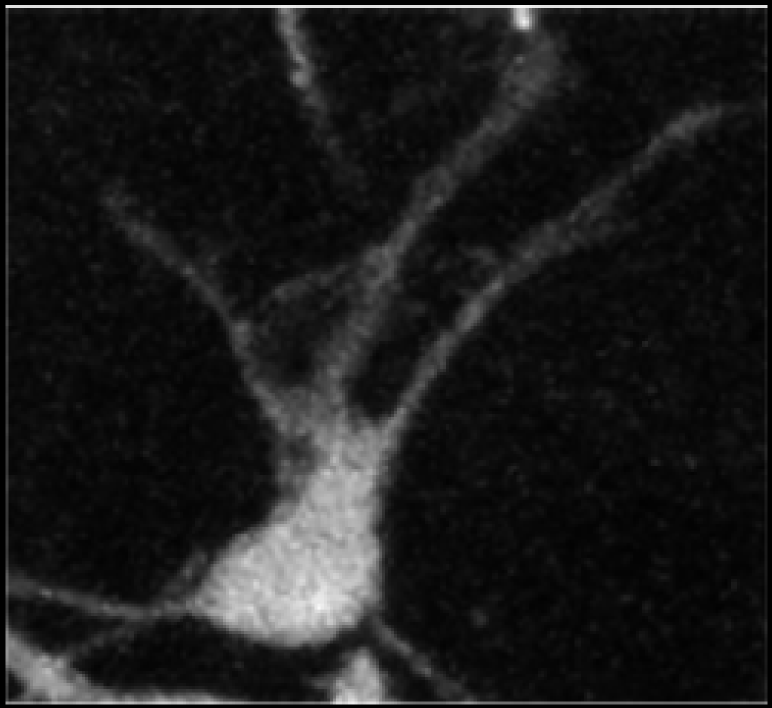
Curvature as regularization in segmentation



[El-Zehiry, 2010]

Data + Curvature term

Curvature as regularization in segmentation



[El-Zehiry, 2010]

Data + Curvature term

Completion property

Curvature as regularization in segmentation

Curvature as regularization in segmentation

$$E(u) = \int_{\Omega} (u-g)^2 dx + \int_{\partial u} \alpha + \beta\kappa^2 ds$$

Curvature as regularization in segmentation

$$E(u) = \int_{\Omega} (u-g)^2 dx + \overbrace{\int_{\partial u} \alpha + \beta \kappa^2 ds}^{\text{Elastica energy}}$$

Curvature as regularization in segmentation

Elastica energy

$$E(u) = \int_{\Omega} (u-g)^2 dx + \int_{\partial u} \alpha + \beta\kappa^2 ds$$

Non-convex term

Curvature as regularization in segmentation

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Elastica energy

Integration domain is unknown

Non-convex term

Curvature as regularization in segmentation

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Non-convex term

Difficult to optimize

Curvature as regularization in segmentation

Elastica energy

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Integration domain is unknown

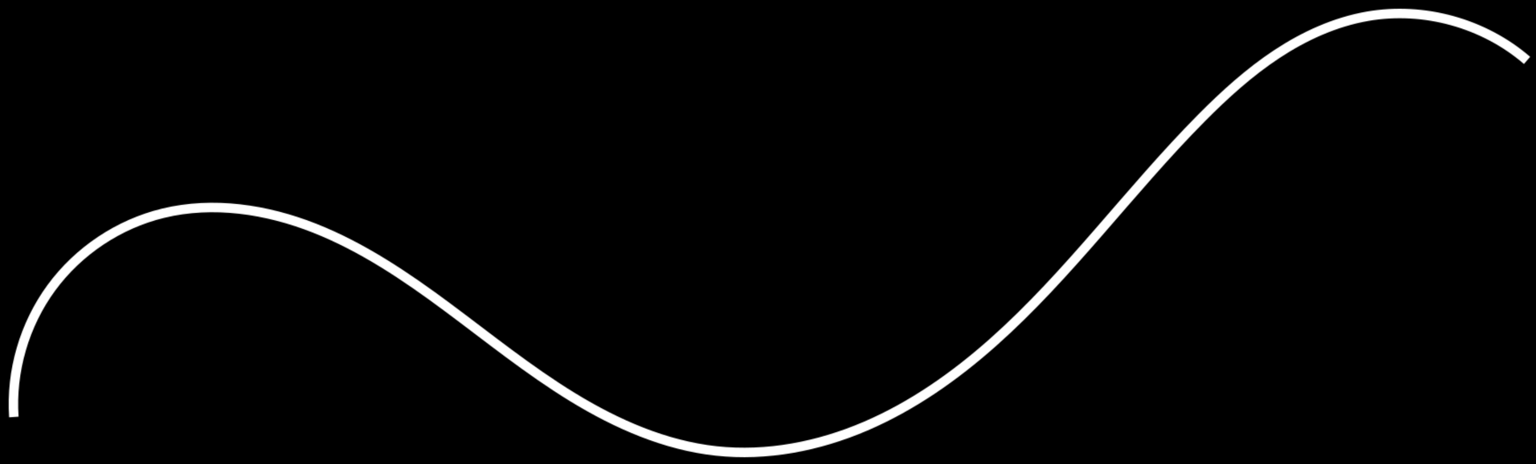
Non-convex term

Difficult to optimize

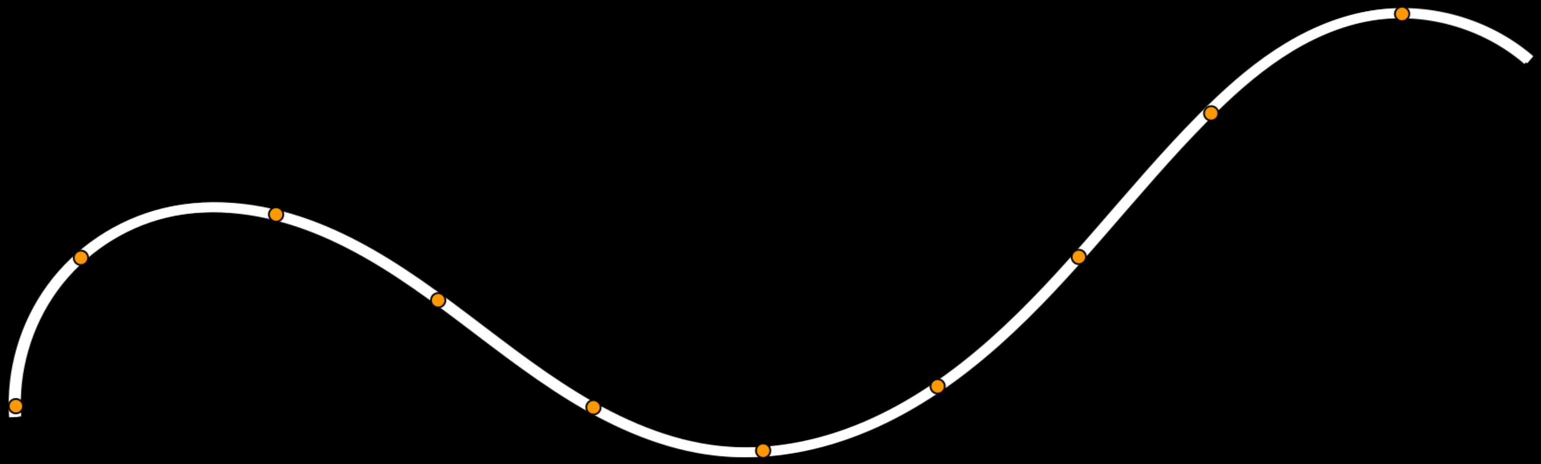
Second order term. Should be careful with discretization scheme

Curvature discretization

Curvature discretization

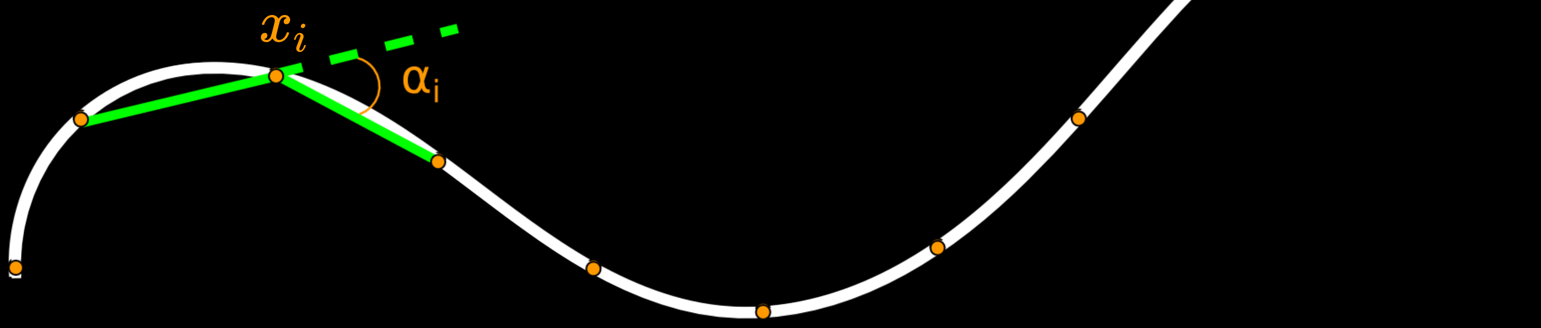


Curvature discretization



Curvature discretization

$$\hat{\kappa}(x_i) = \frac{\alpha_i}{\min\{l_i, l_{i+1}\}}$$



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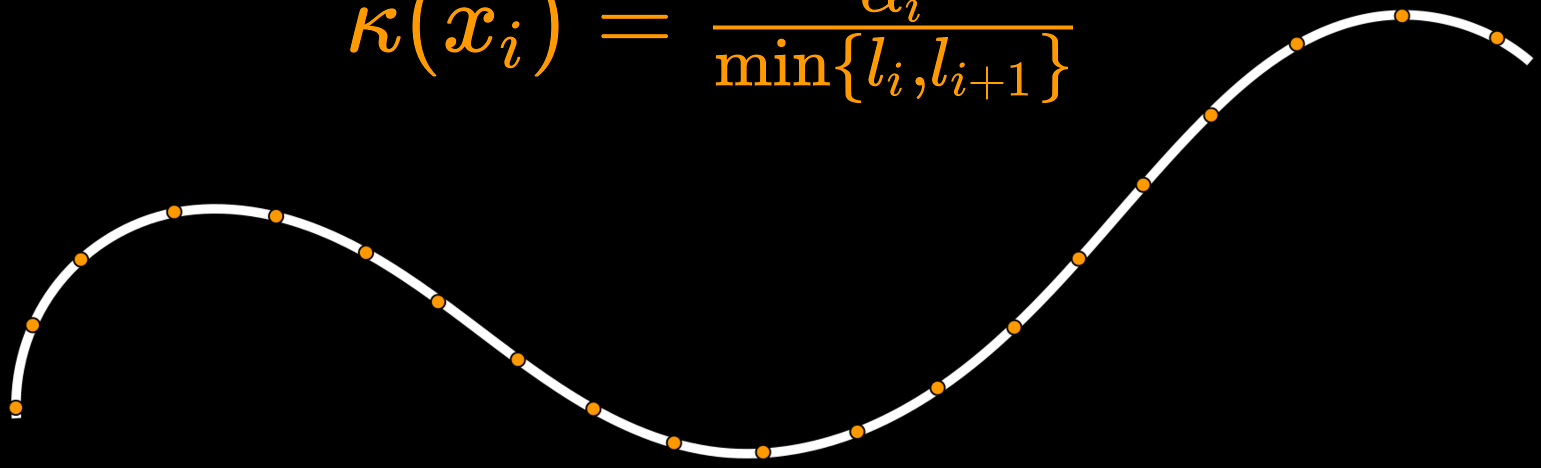

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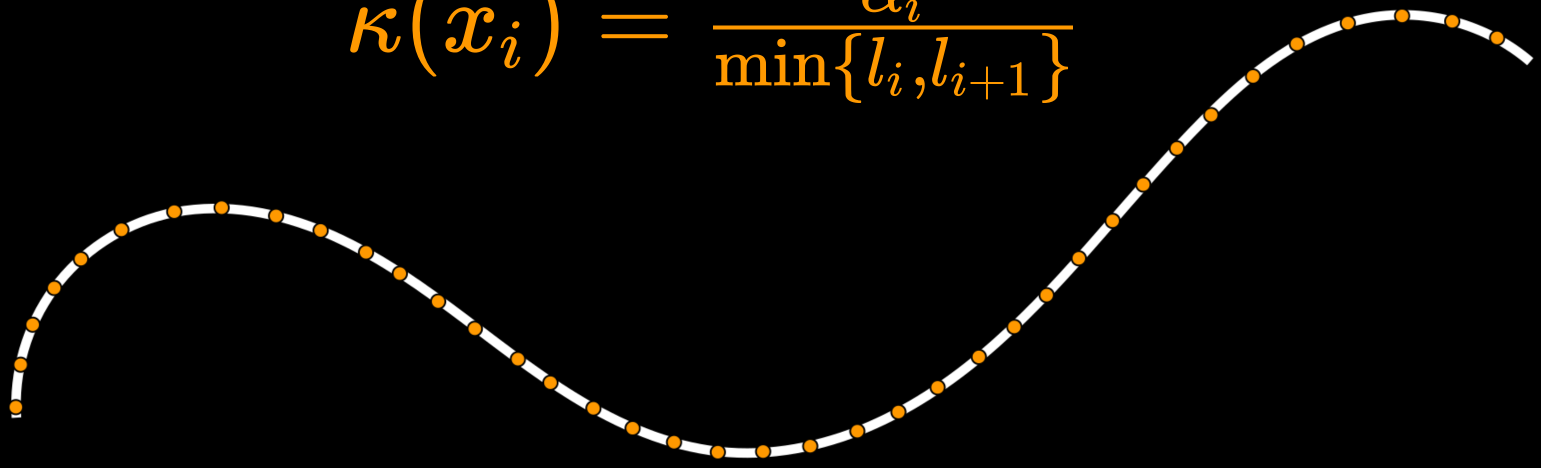

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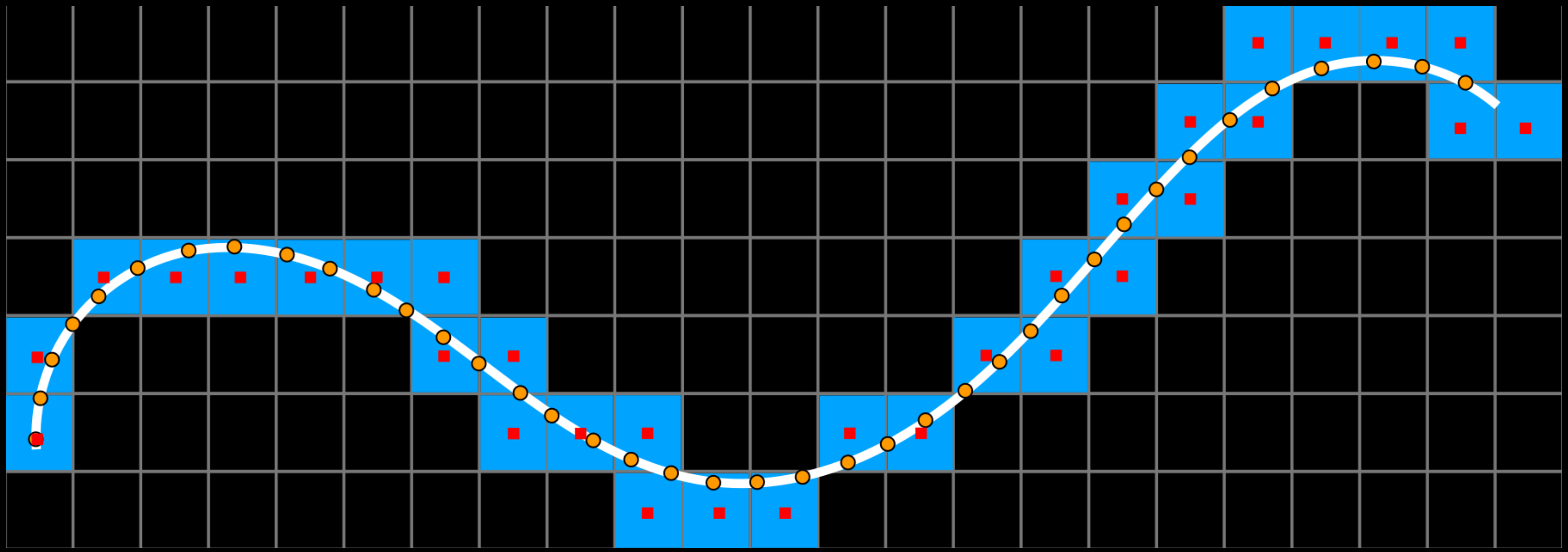

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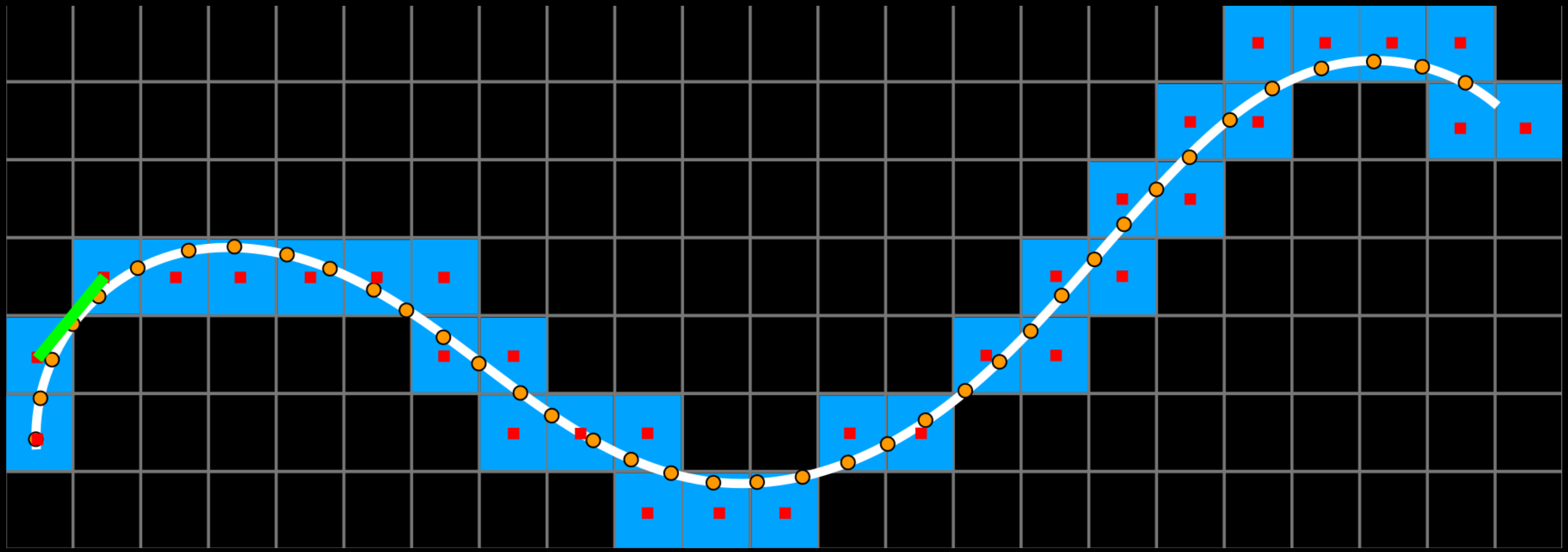

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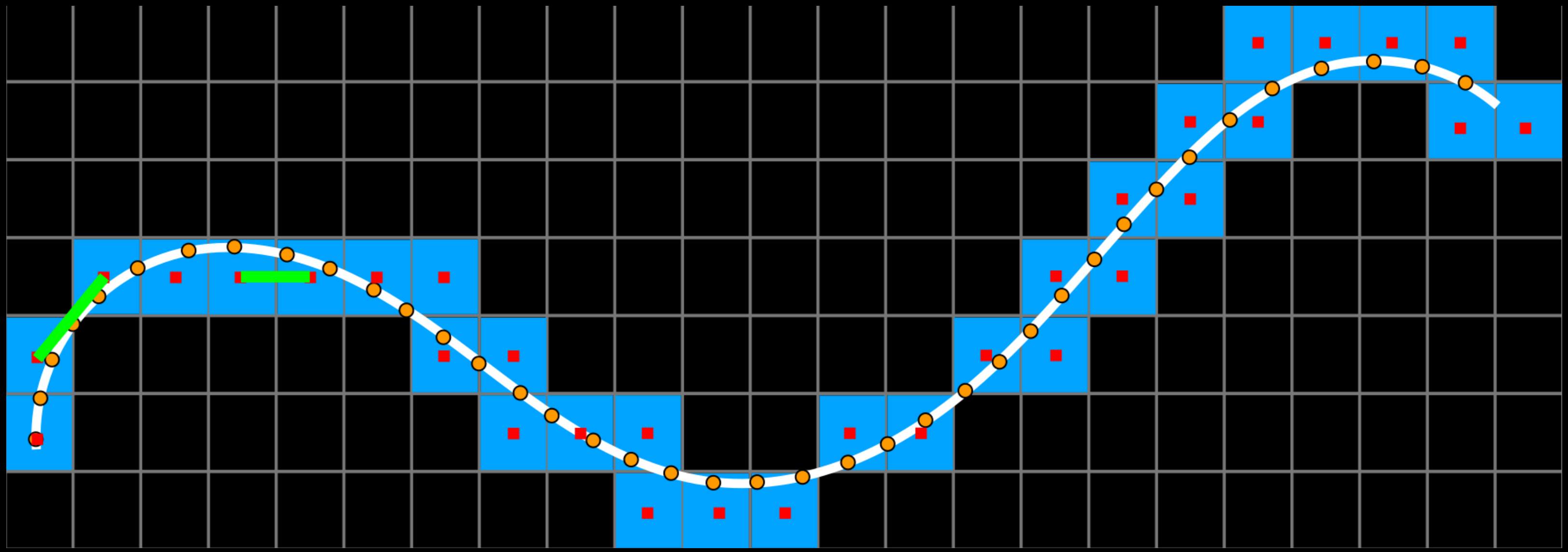
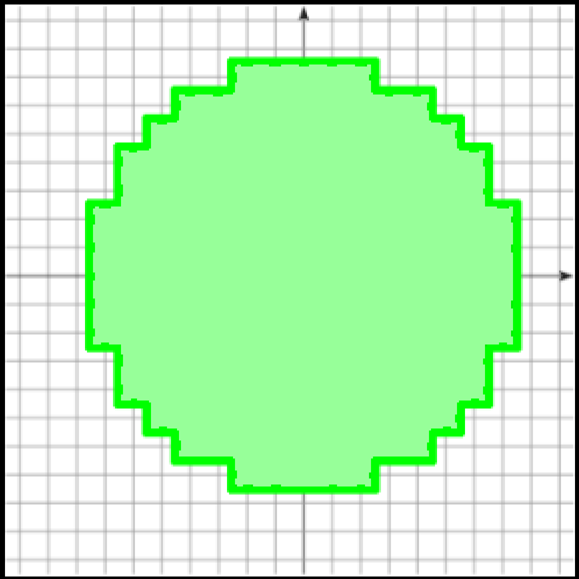


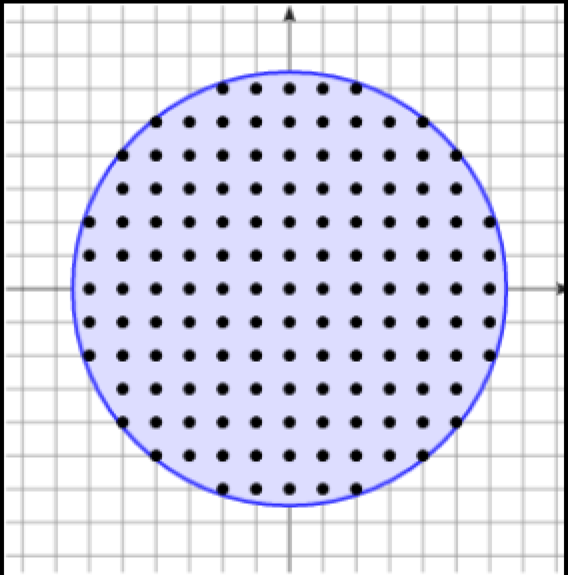
Digitization ambiguity

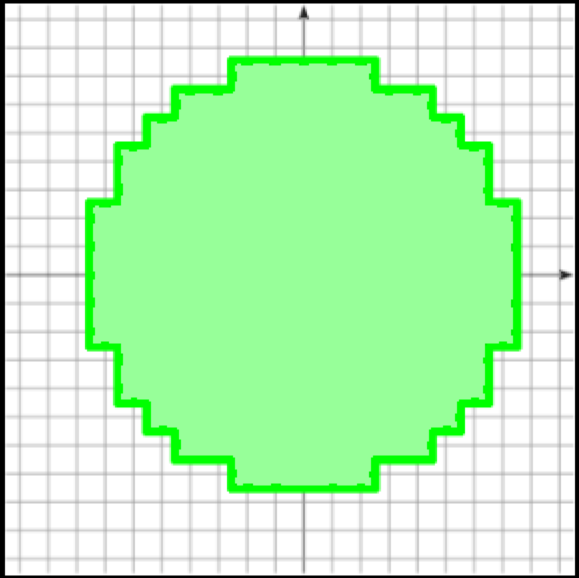
Digitization ambiguity



[Roussillon, Lachaud 2011]

Digitization ambiguity





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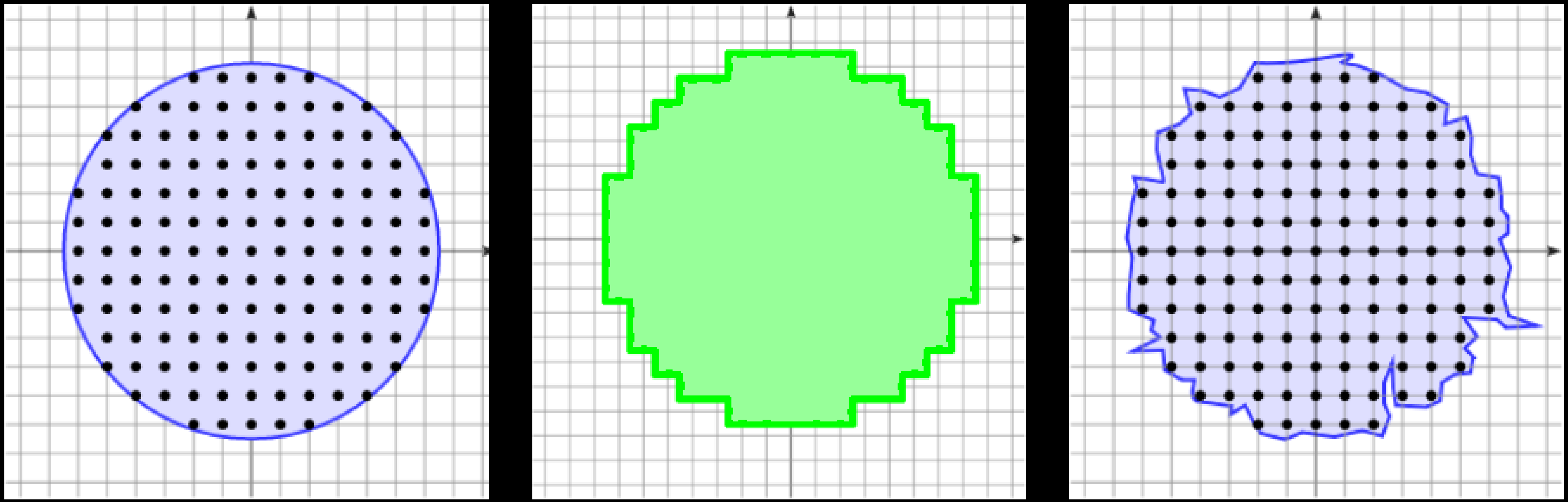
Digitization ambiguity



[Roussillon, Lachaud 2011]

Multigrid Convergence

Multigrid Convergence

The estimated quantity $\hat{E}(D_h(X), \hat{x}, h)$ gets arbitrarily close from $E(X, x)$ as resolution increases.

Integral based estimator for curvature

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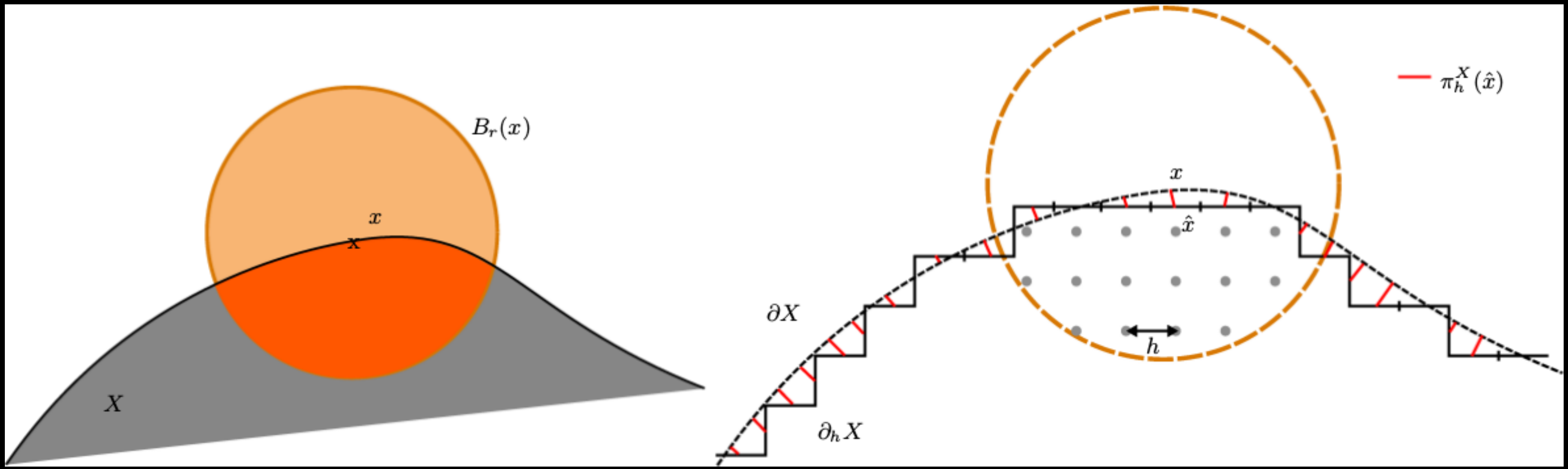

[Coeurjolly, Lachaud, Levallois 2013]

$$\hat{\kappa}_{R,h}(x_i) = \frac{3}{R^3}\left(\frac{\pi R^2}{2} - \widehat{Area}(B_{R,h}(x_i) \cap D_h)\right)$$

To sum up

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We want to use curvature as regularization term in models of image processing tasks

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We believe that by using a multigrid convergent estimator, we can recover better results

Notation

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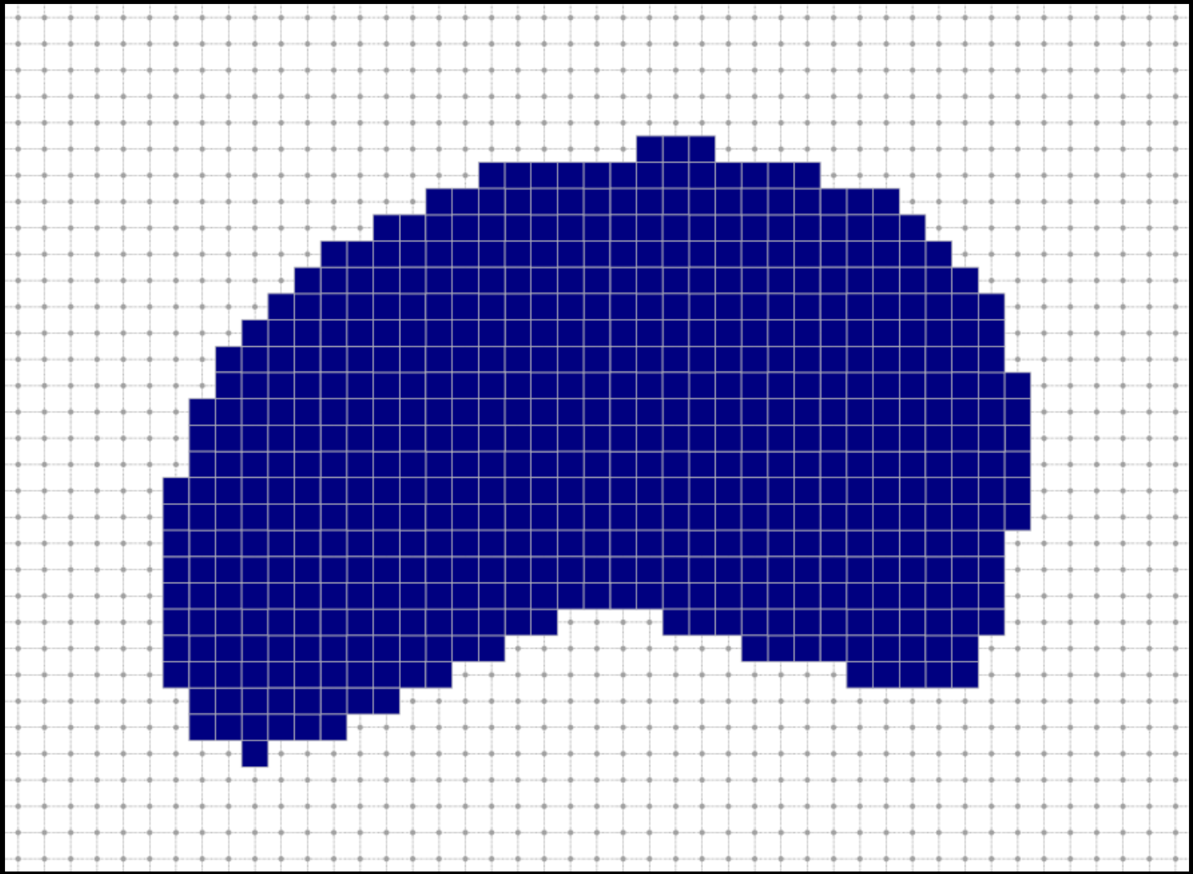
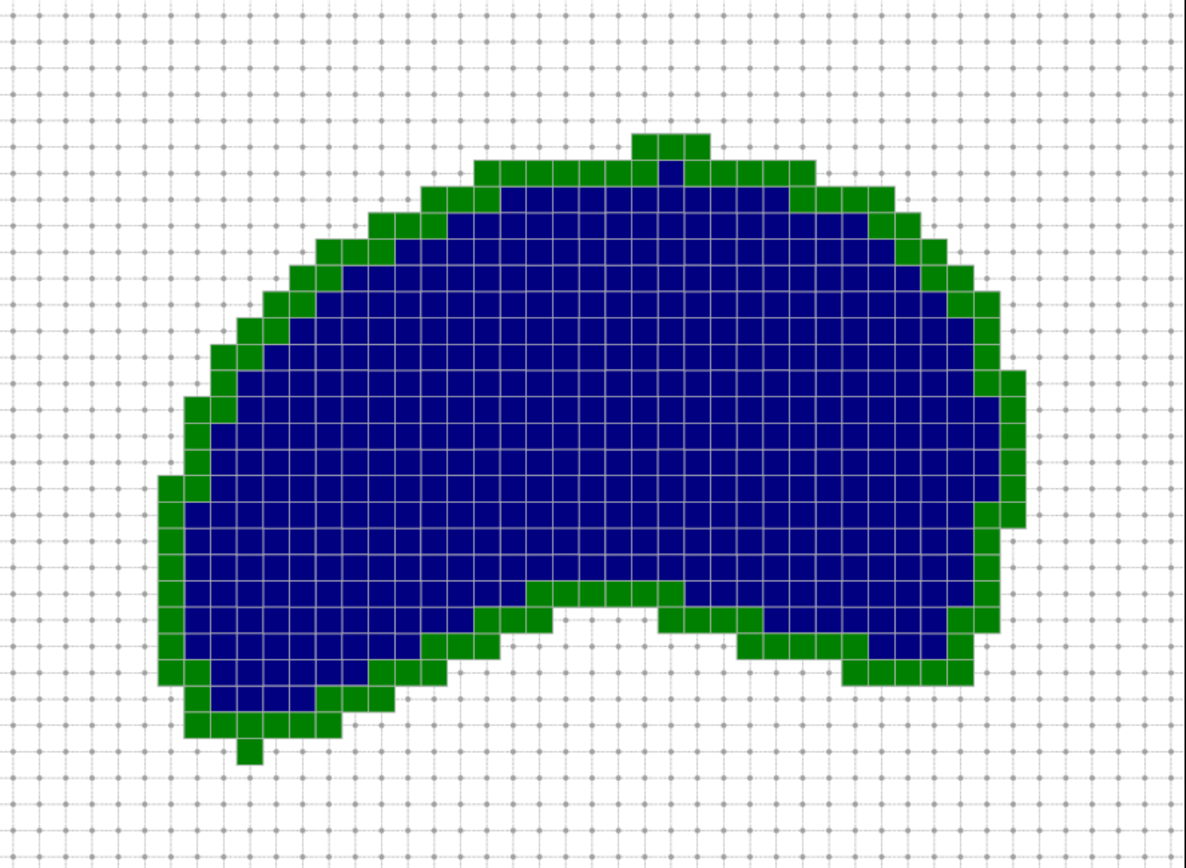
Let D be a connected digital shape



Notation

Let D be a connected digital shape

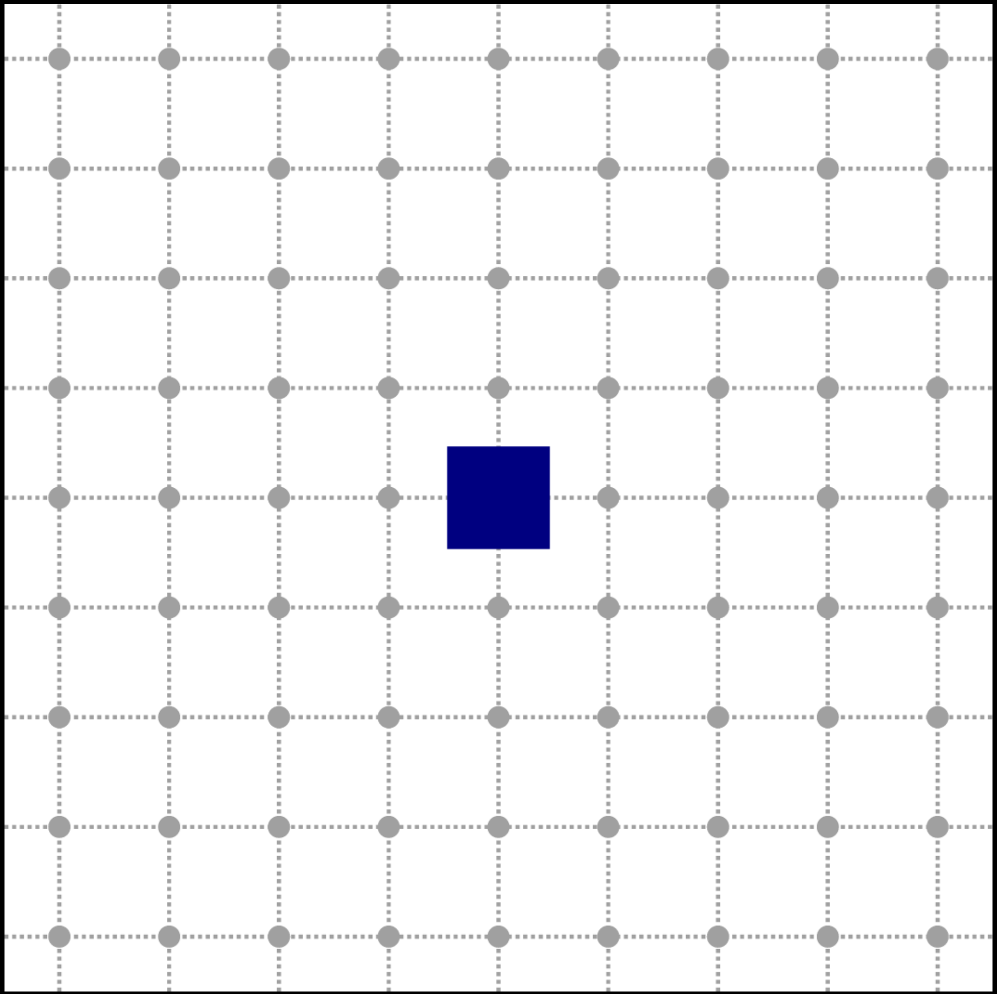


$\mathcal{C}(D)$

4-connected pixel boundary

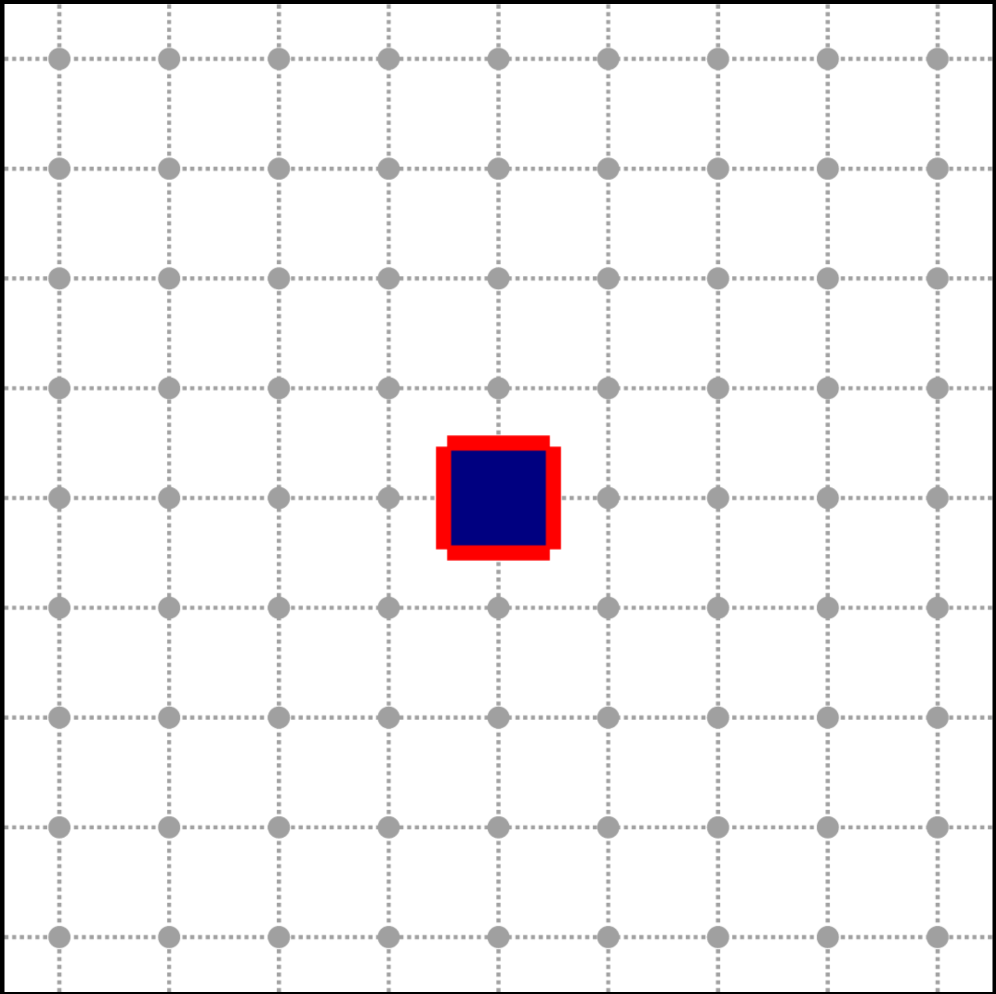
Cellular grid model

Cellular grid model



Cells (pixels)

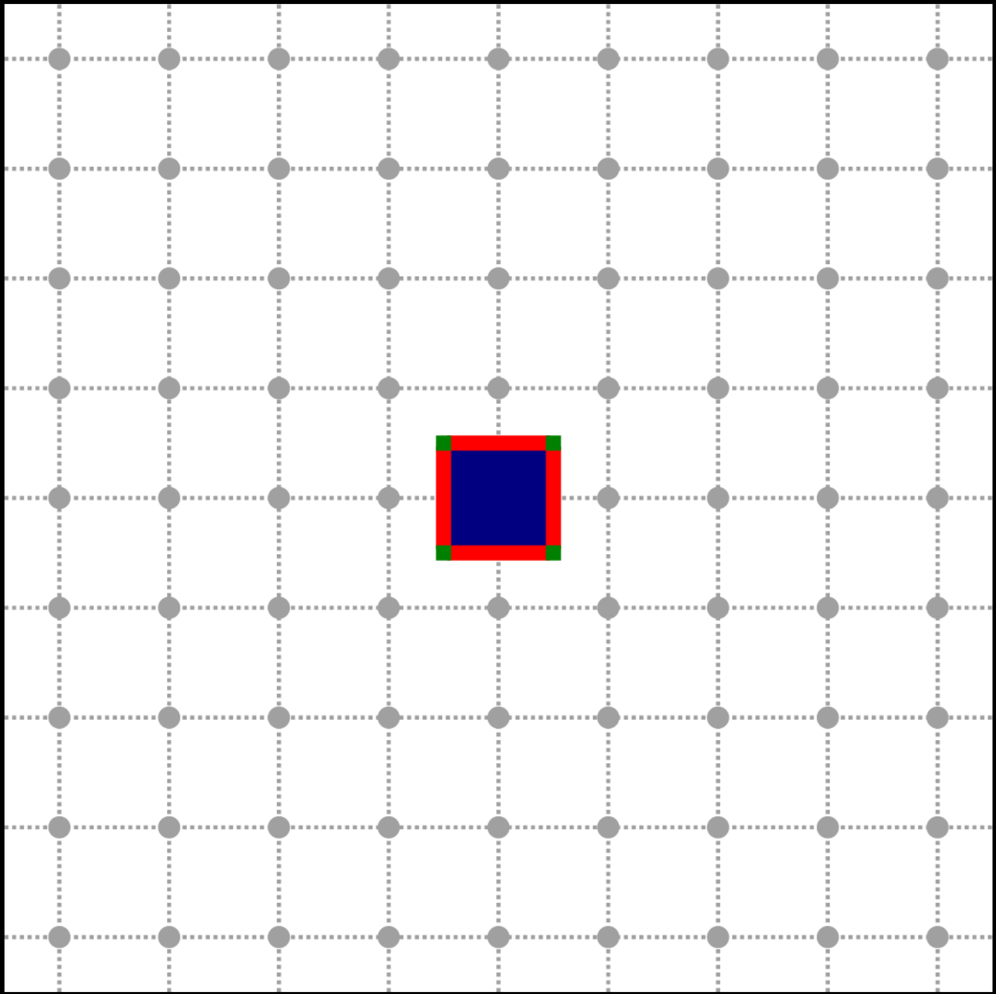
Cellular grid model



Cells (pixels)

Linels

Cellular grid model



Cells (pixels)

Linels

Pointels

Curve evolution model

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$$E(u) = \int_{\partial u} \kappa^2 ds$$

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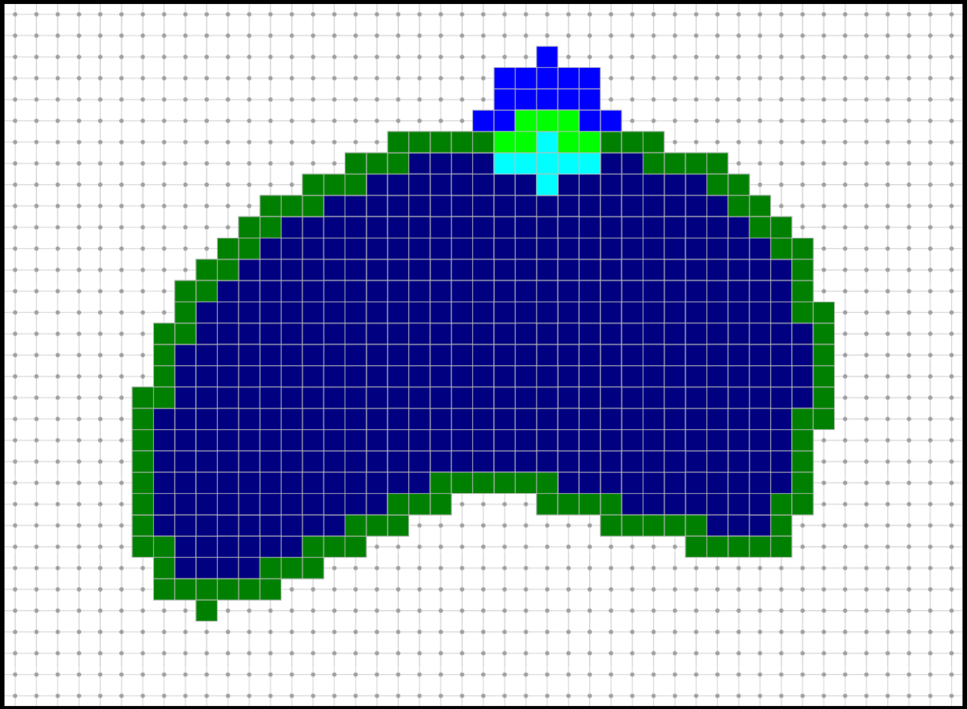
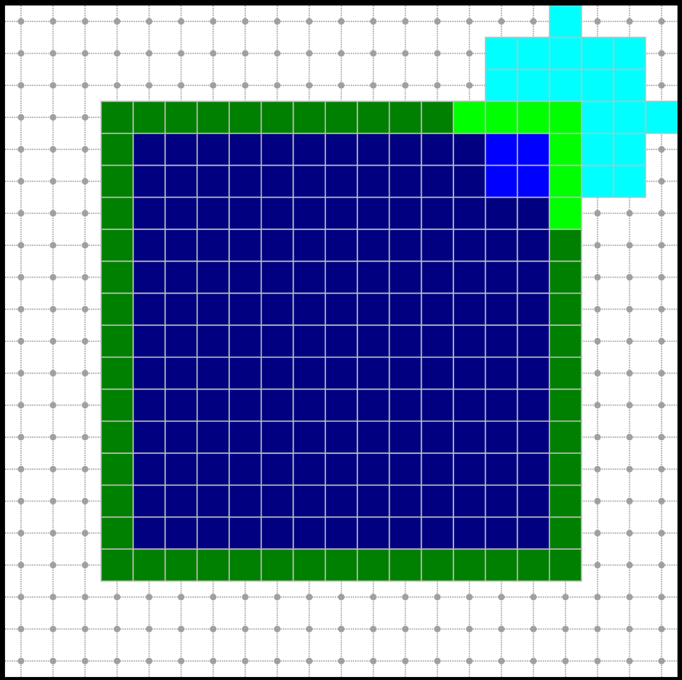


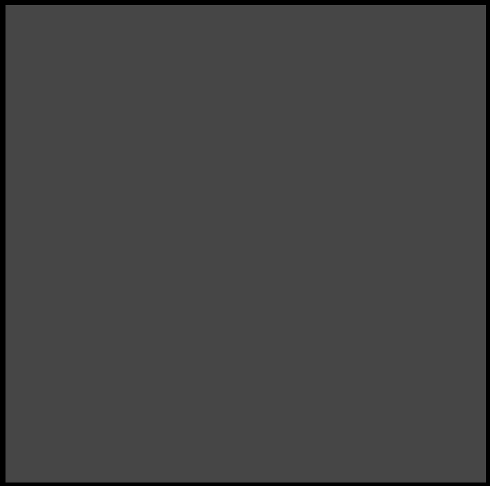
$$\sum_{y_i \in \mathcal{C}(D)} \hat{k}_R^2(y_i)$$

Curve evolution model

Curve evolution model

$$\min_{y} \sum_{y_i \in \mathcal{C}(D)} \hat{k}^2_R(y_i), \quad y \in \{0,1\}^{|\mathcal{C}(D)|}$$





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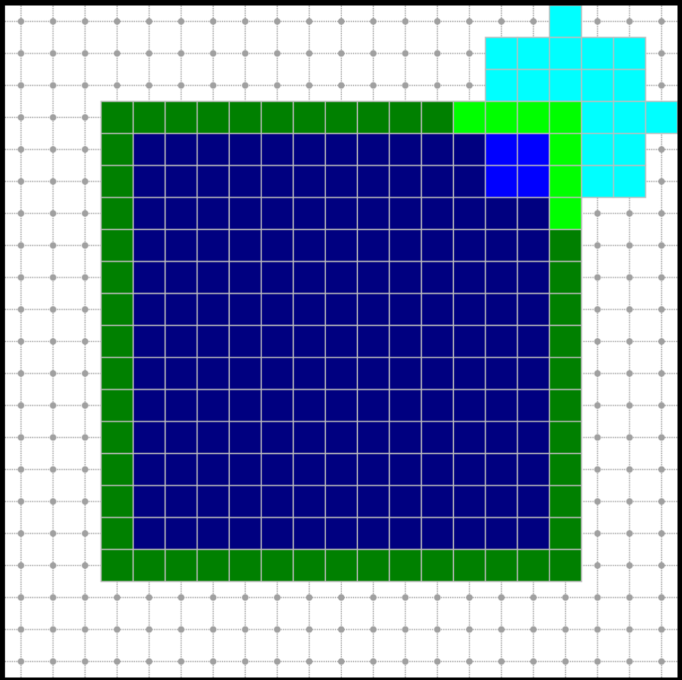

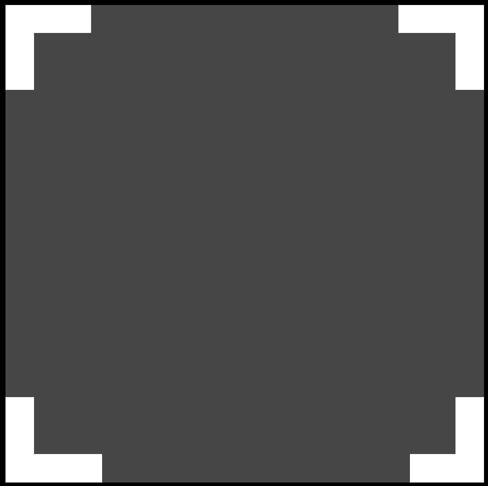

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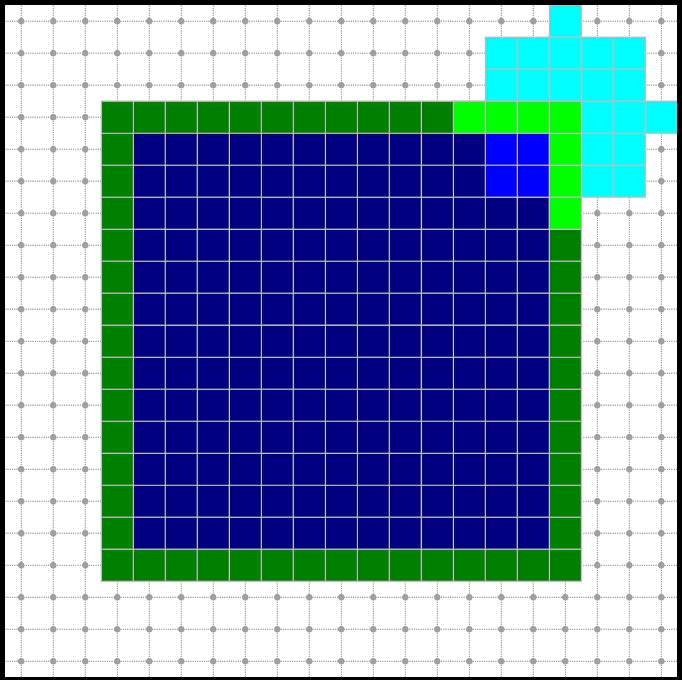
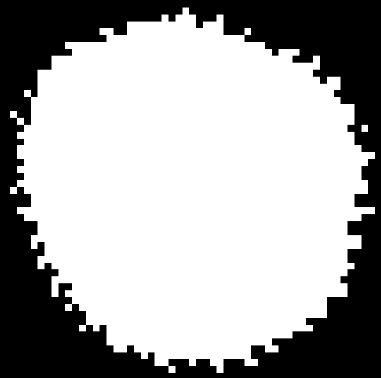


A first evolution

A first evolution



Sensible regions indication

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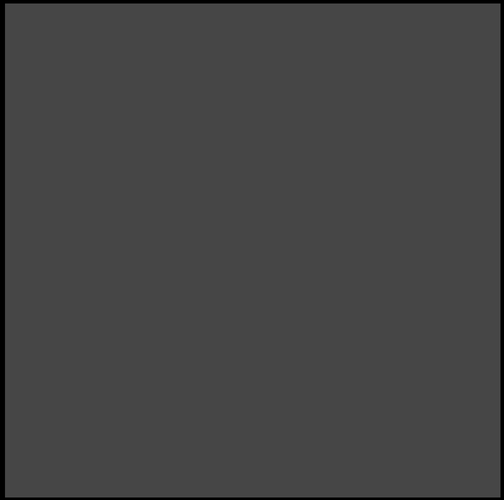

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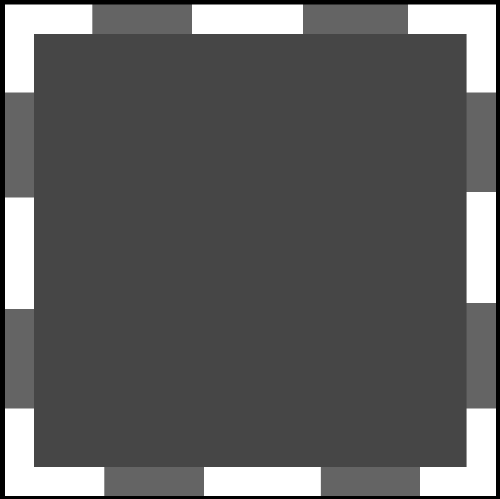

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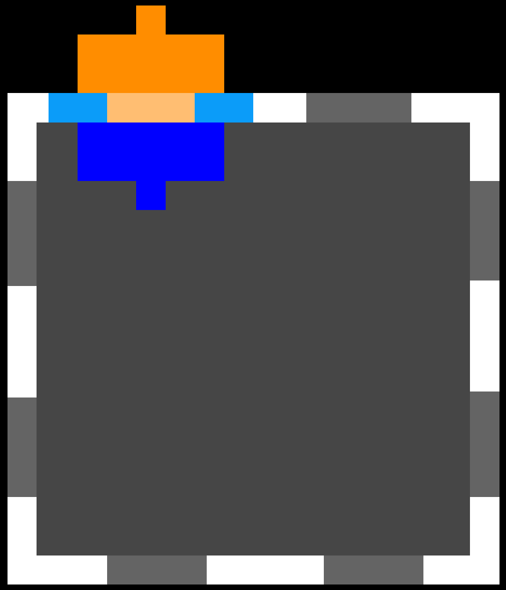

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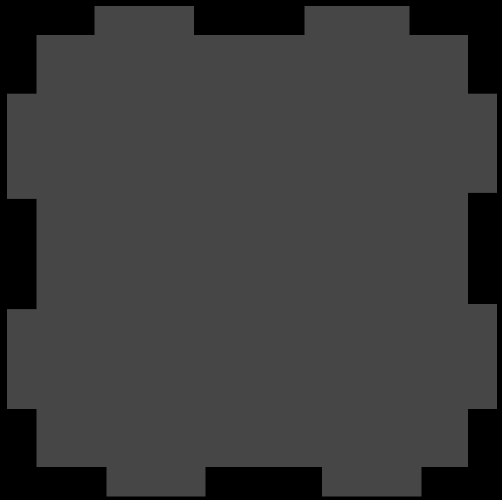
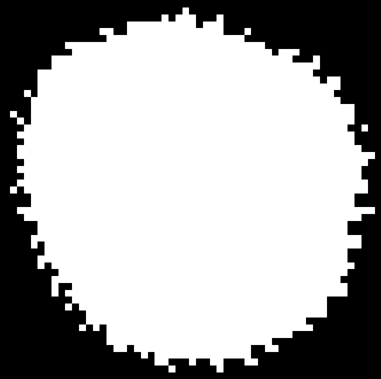


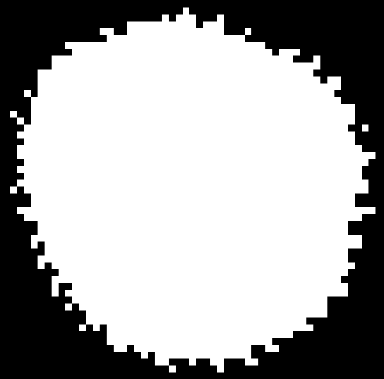
Perimeter penalization

Perimeter penalization



$$\min_{y} \sum_{y_i \in \mathcal{O}} \hat{k}_R^2(y_i)$$

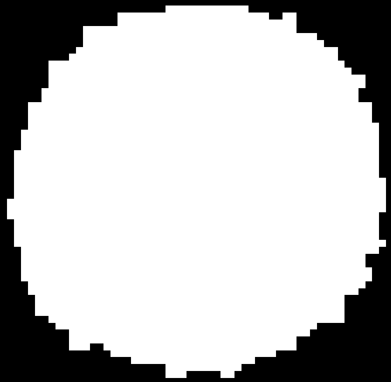
Perimeter penalization



$$\min_{y} \sum_{y_i \in \mathcal{O}} \hat{k}_R^2(y_i)$$

$$\min_{y} \alpha \sum_{y_i \in \mathcal{O}} \hat{k}_R^2(y_i) + \beta \sum_{y_i \in \mathcal{O}} \sum_{y_j \in \mathcal{N}_4(y_i)} (y_i - y_j)^2$$

Perimeter penalization



$\alpha = 1$

$\beta = 0.5$

$$\min_y \sum_{y_i \in \mathcal{O}} \hat{k}_R^2(y_i)$$

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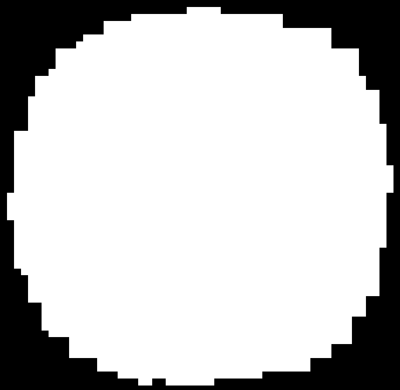

$$\alpha = 1$$
$$\beta = 1.0$$

$$\min_y \sum_{y_i \in \mathcal{O}} \hat{k}_R^2(y_i)$$

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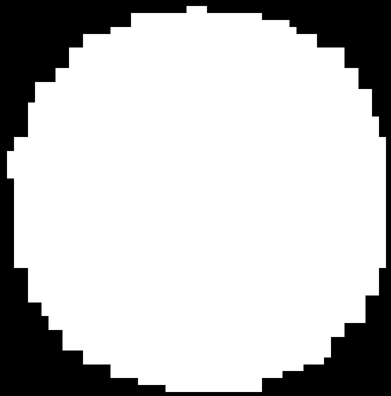

$$\alpha = 1$$

$$\beta = 2.0$$

$$\min_{y} \sum_{y_i \in \mathcal{O}} \hat{k}_R^2(y_i)$$

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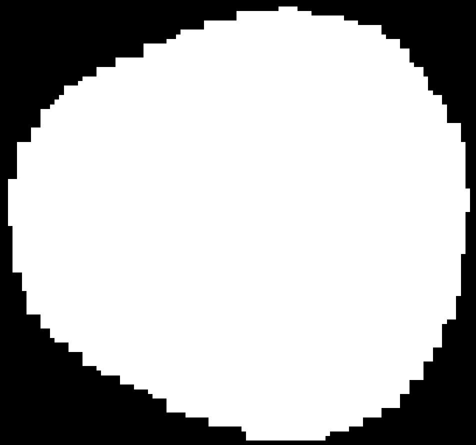

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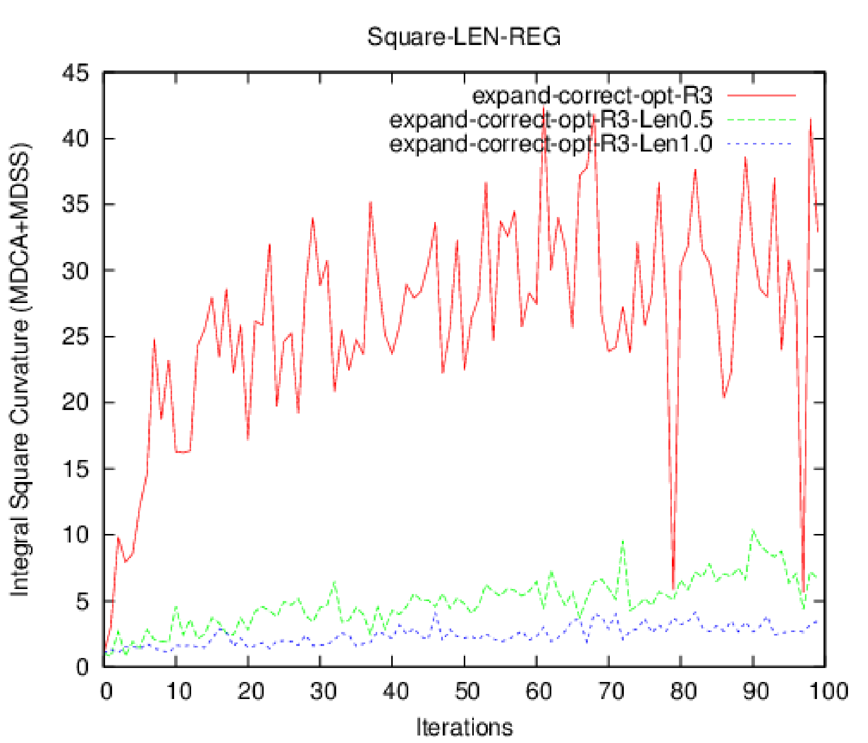

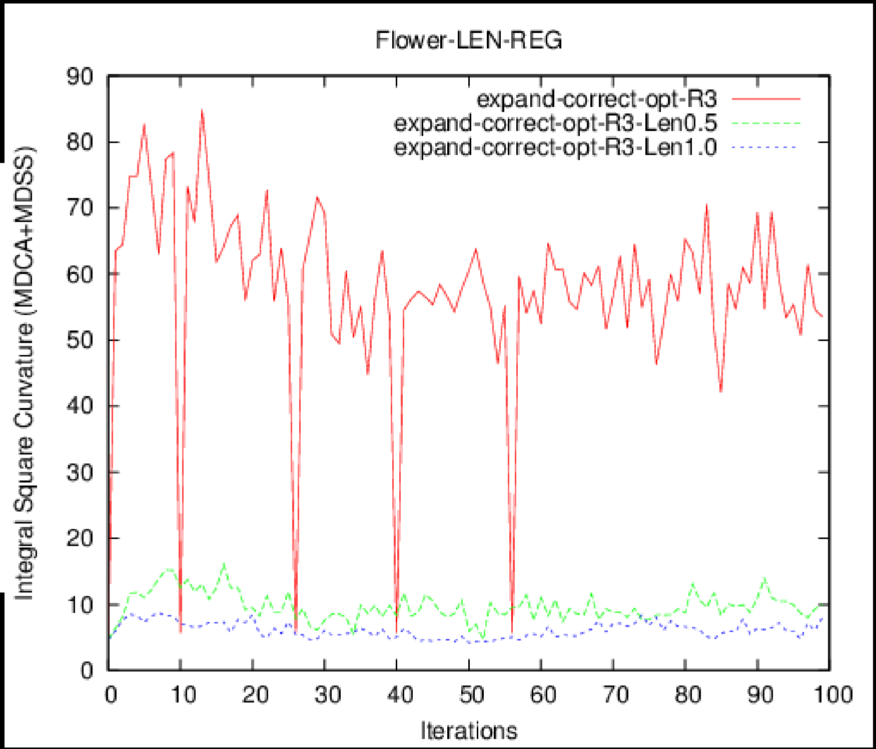

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Filtering artifacts

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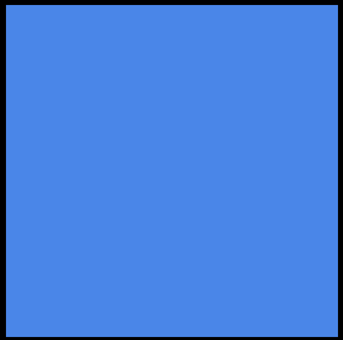
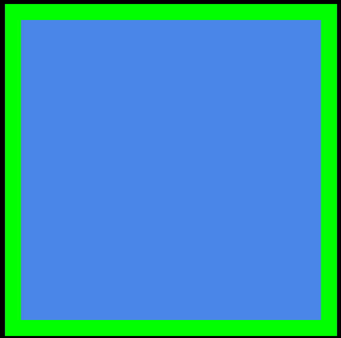


Filtering artifacts

Optimization region $\mathcal{O}$



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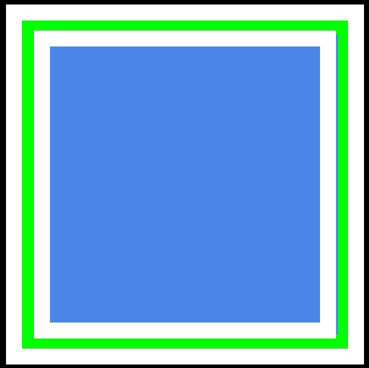
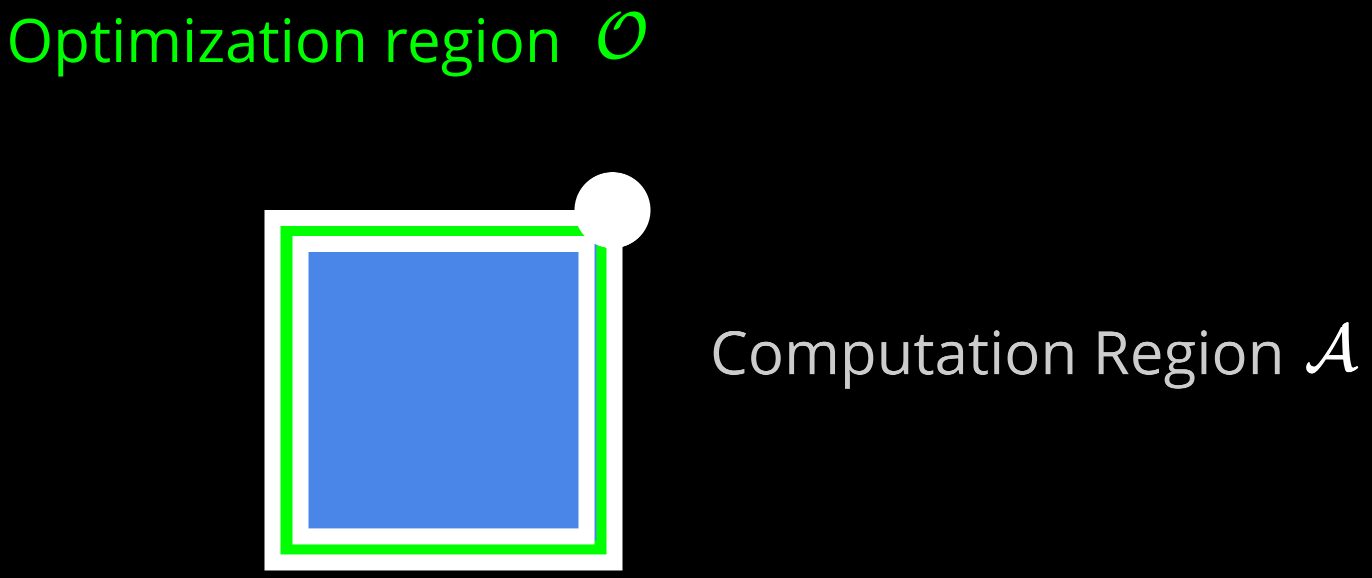


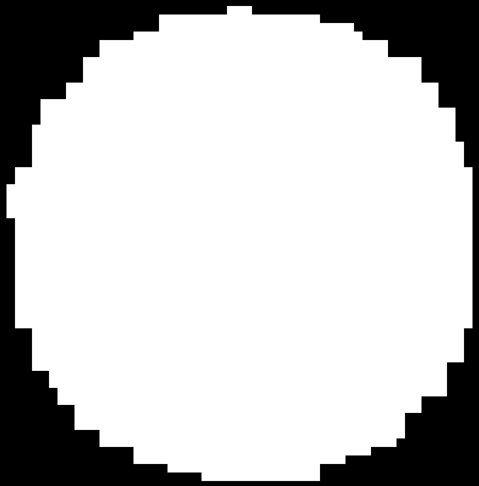
Computation Region $\mathcal{A}$

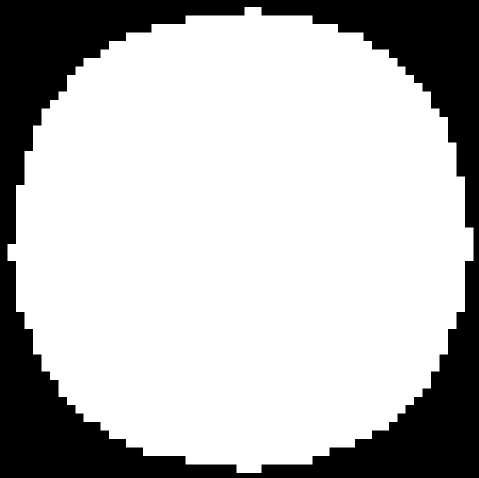
Filtering artifacts



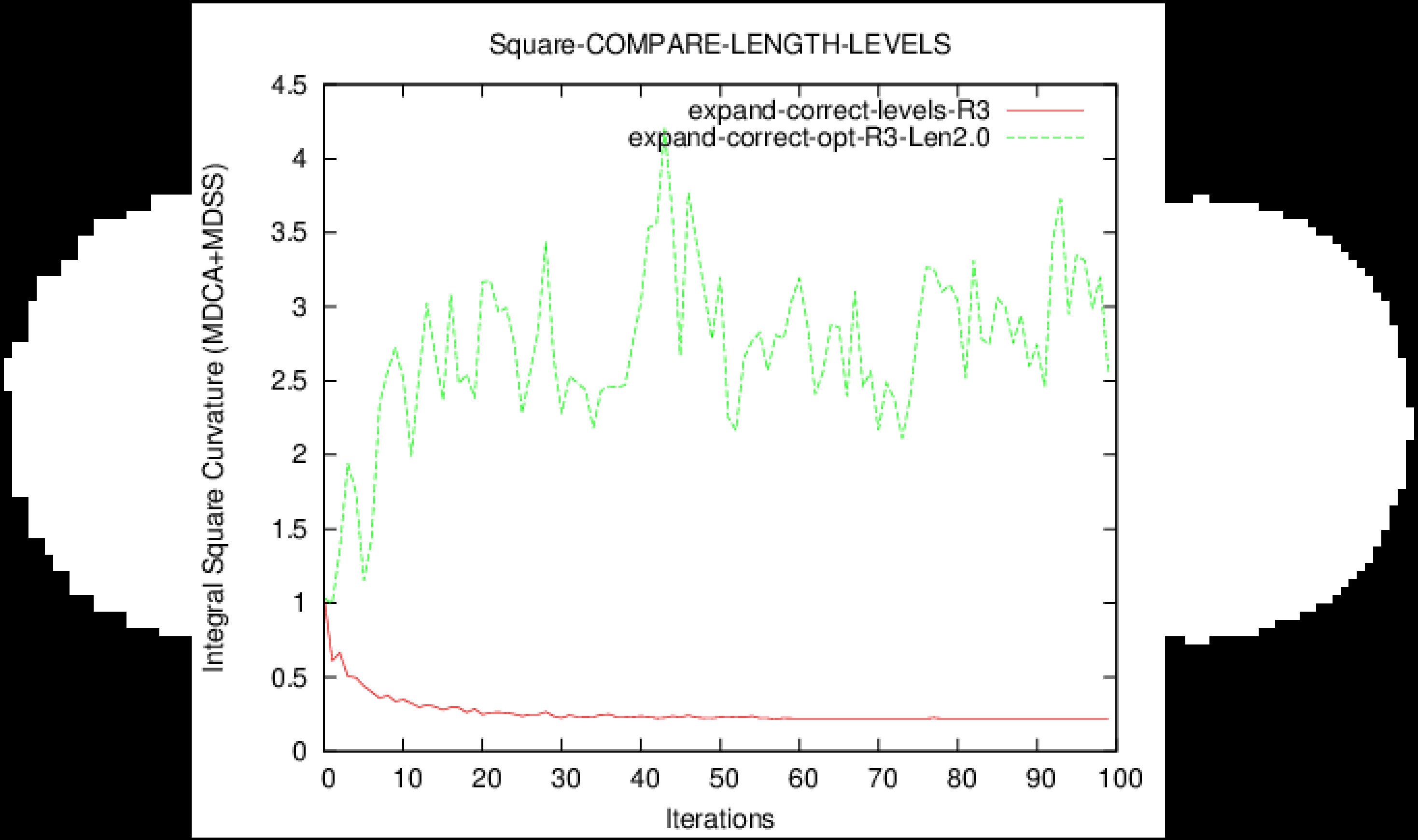
$$\min_{y} \sum_{a_i \in \mathcal{A}(D)} \hat{k}_R^2(a_i, y), \quad y \in \{0,1\}^{|\mathcal{C}(D)|}$$

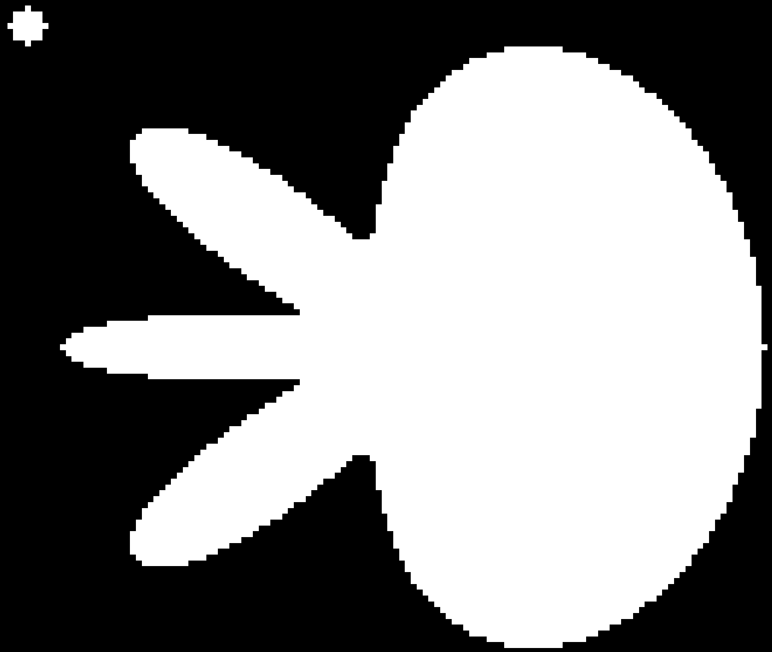
Bands evolution



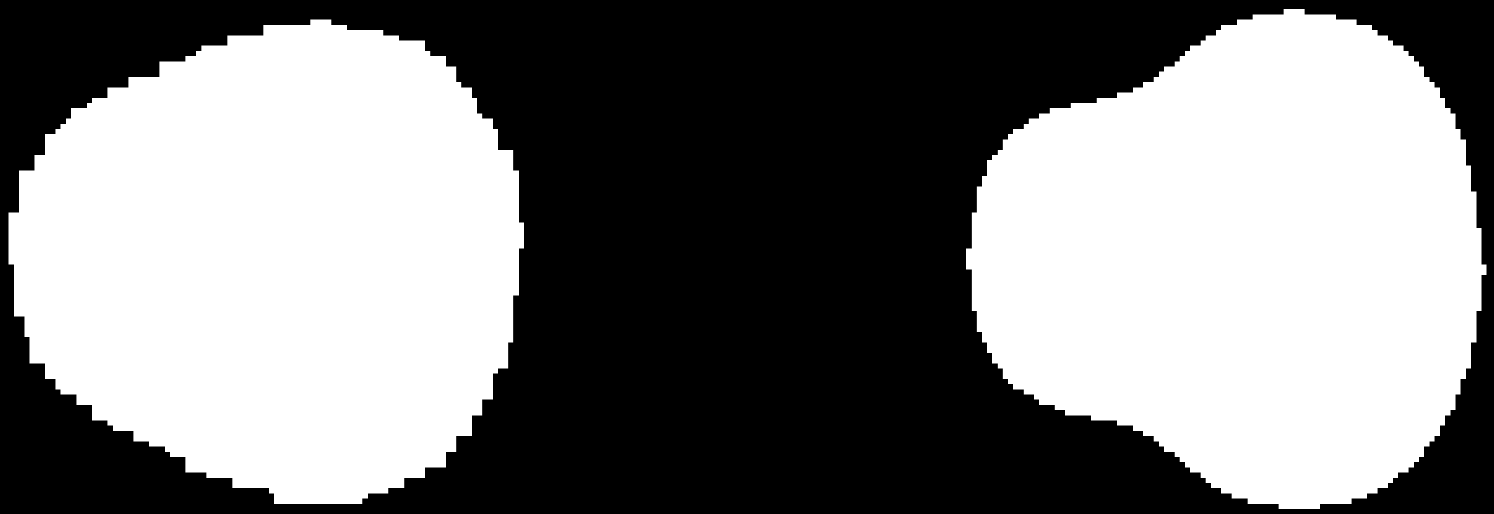


Bands evolution





Bands evolution



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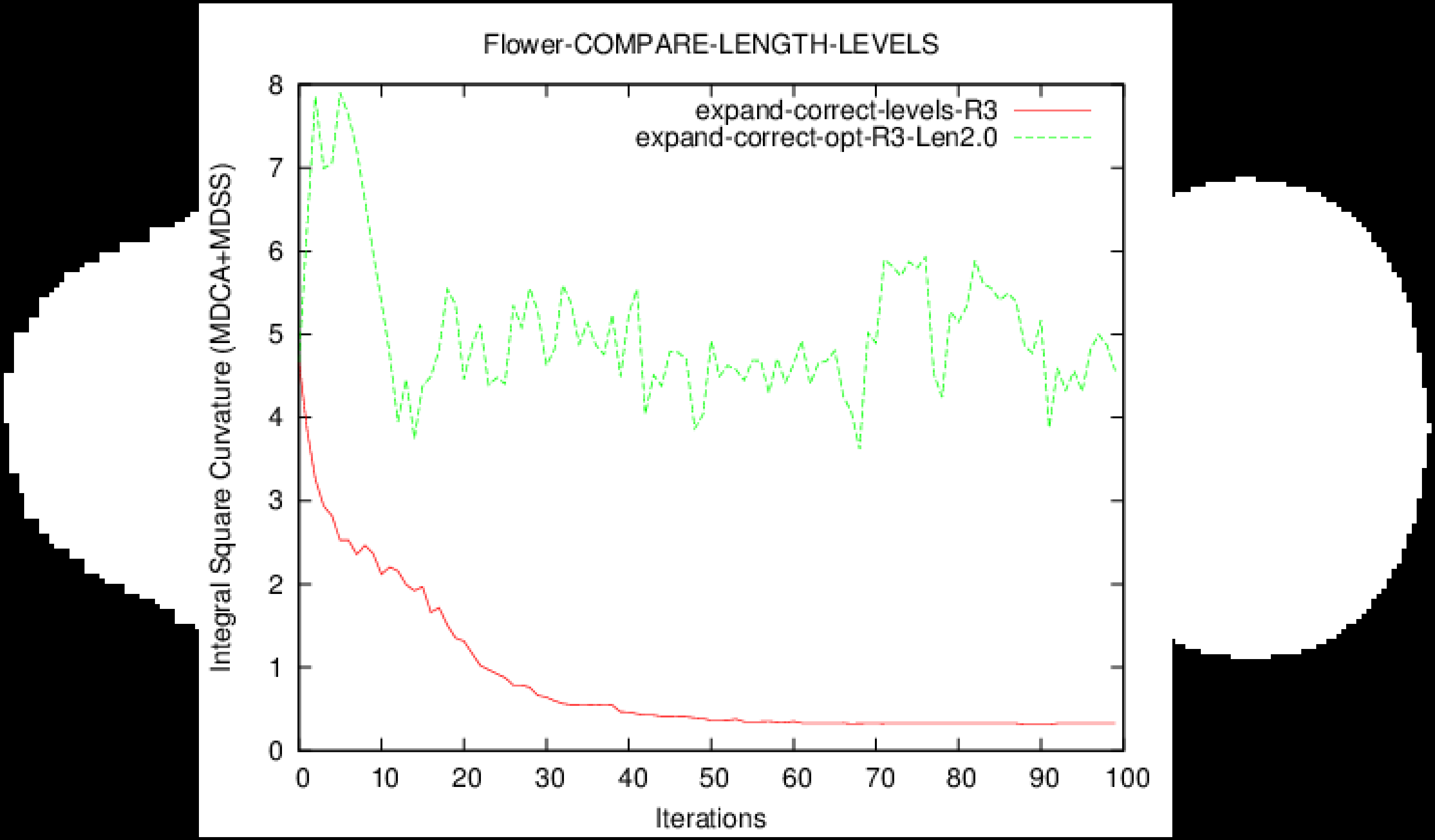
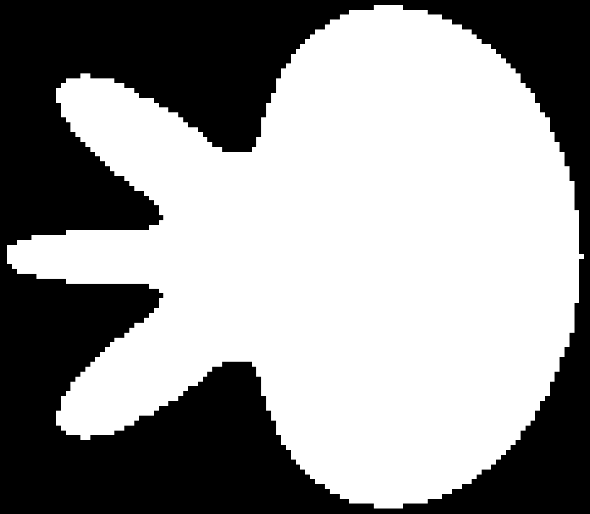
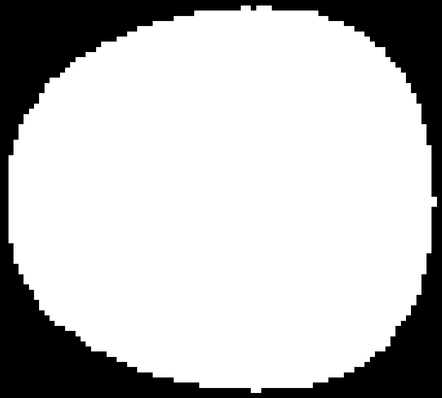


Ball radius effect

$R = 3$

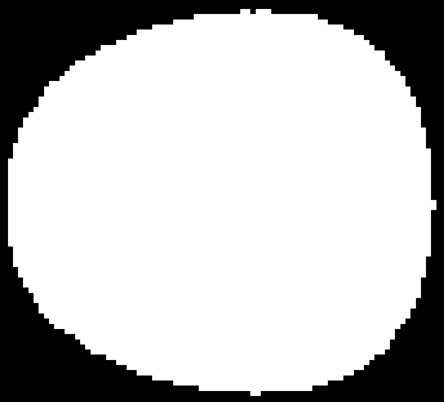


$R = 5$

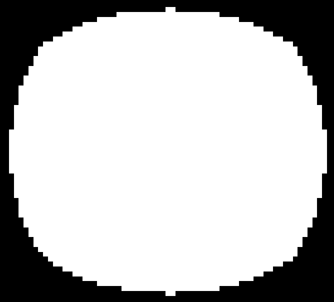


Ball radius effect

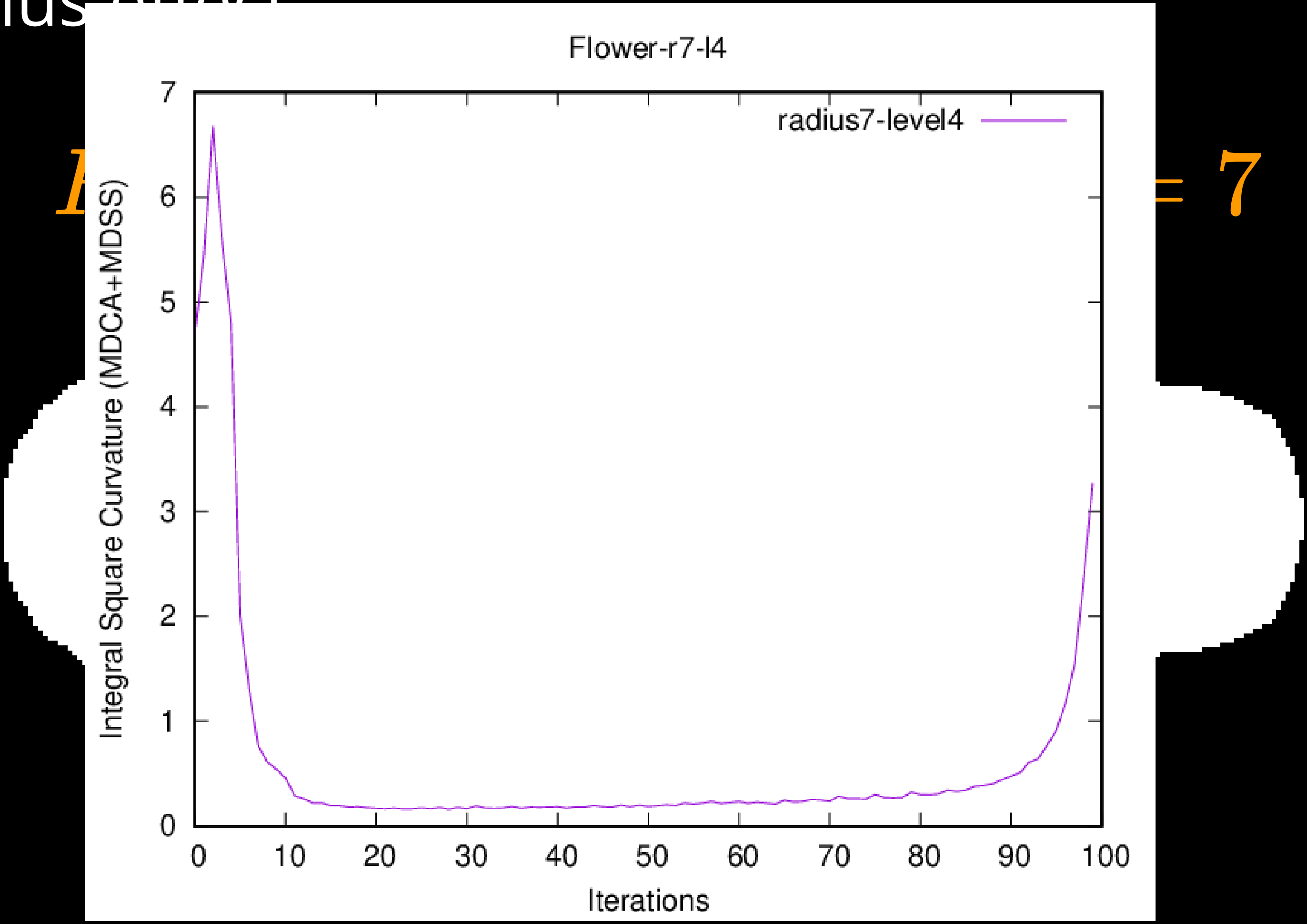
$R = 5$



$R = 7$



Ball radius effect



Quadratic pseudo-boolean function

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$$\min_{y} \sum_{y_i \in \mathcal{C}(D)} \hat{k}^2_R(y_i)$$

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$$\min_{y} \sum_{p \in \mathcal{C}(D)} \left((1/2 + |F_r(p)| - c_2) \cdot \sum_{y_i \in Y_r(p)} y_i + \sum_{y_i, y_j \in Y_r(p); i<j} y_i y_j \right)$$

Quadratic pseudo-boolean function

$$\min_{y} \sum_{y_i \in \mathcal{C}(D)} \hat{k}_R^2(y_i)$$

$$\min_{y} \sum_{p \in \mathcal{C}(D)} \left((1/2 + |F_r(p)| - c_2) \cdot \sum_{y_i \in Y_r(p)} y_i + \sum_{y_i, y_j \in Y_r(p); i<j} y_i y_j \right)$$

Supermodular energy

$$\frac{\partial^2 \hat{E}}{\partial y_i \partial y_j} \geq 0, \quad \forall i \neq j$$

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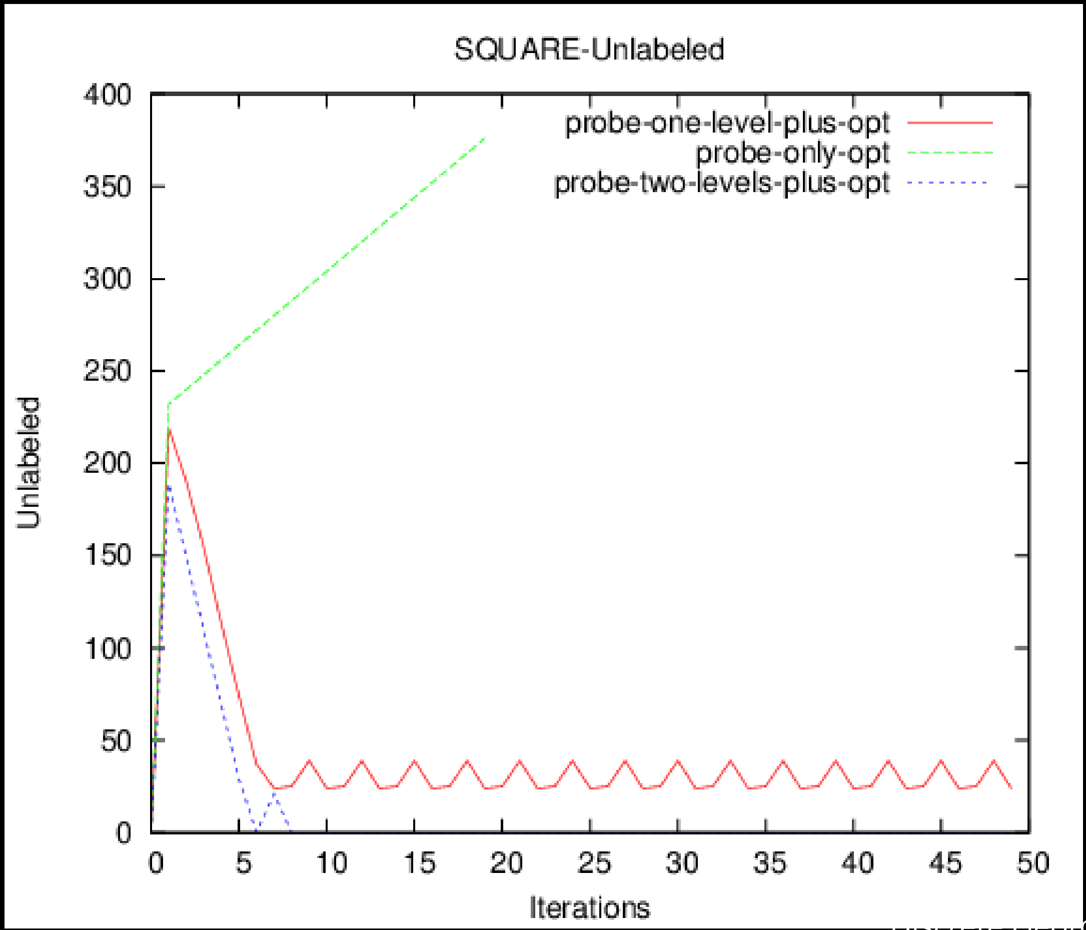
If energy is submodular, **QPBOP** labels all variables.

Unlabeled pixels



Segmentation post-processing

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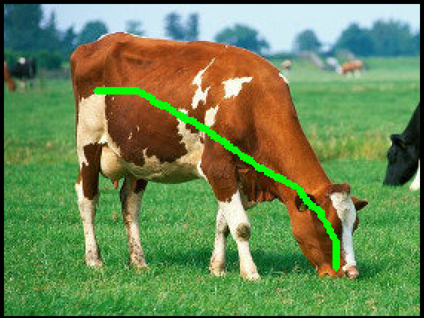

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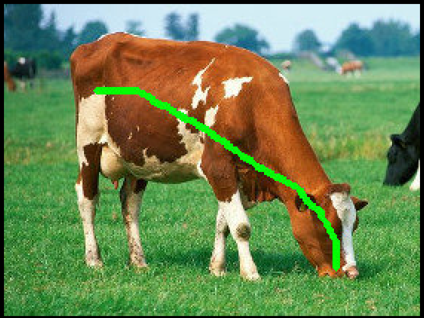

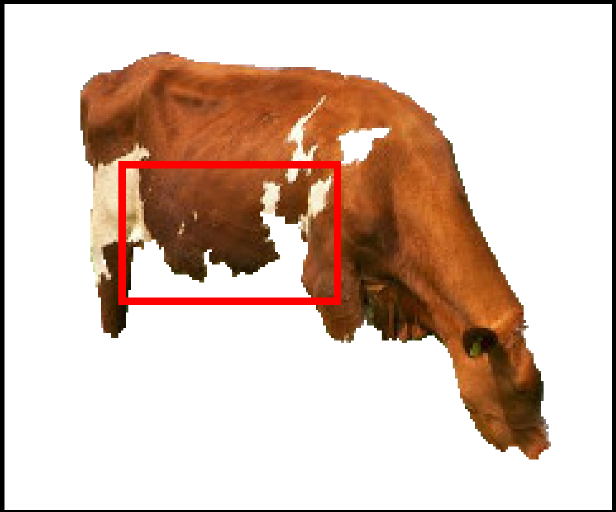

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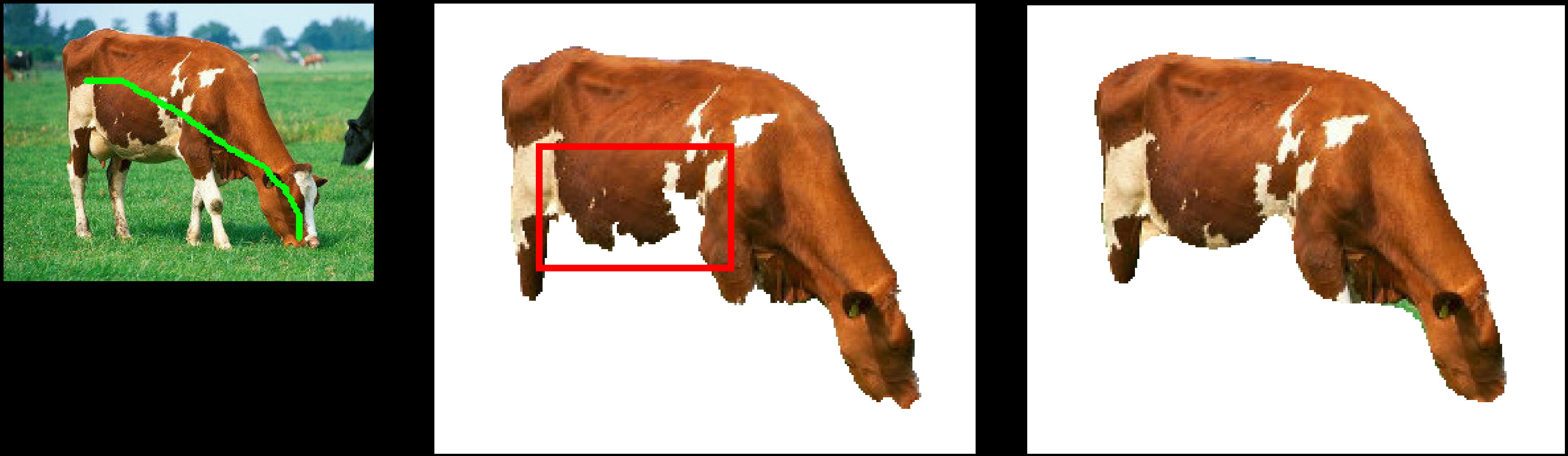
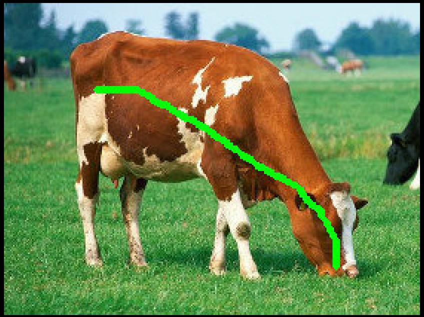
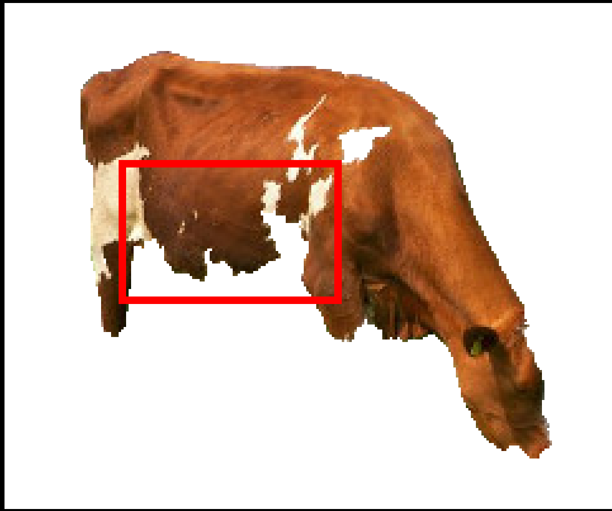


Segmentation post-processing


Segmentation post-processing

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Model summary

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- (+) Flow based on a multigrid convergent estimator of curvature
- (+) Post-processing step in image segmentation
- (+) Works well with extra terms (data fidelity, perimeter)
- (−) Too local. Completion property is not recovered

Digital Curvature Evolution Model for Image Segmentation

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Thank you for your attention!