Towards well-composedness of cell complexes over nD pictures

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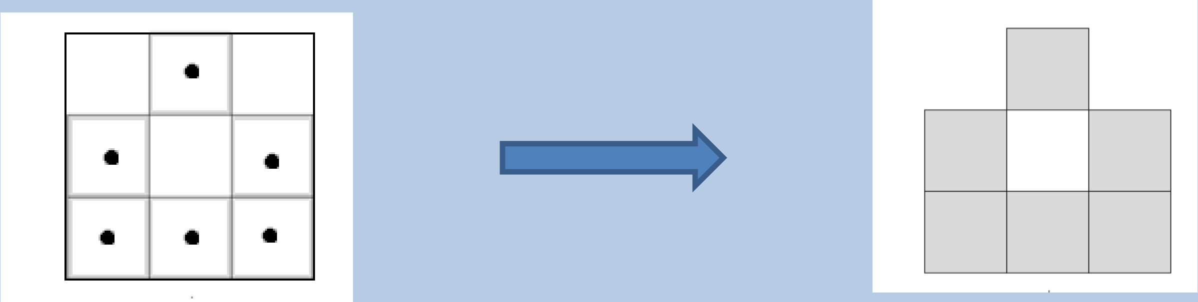
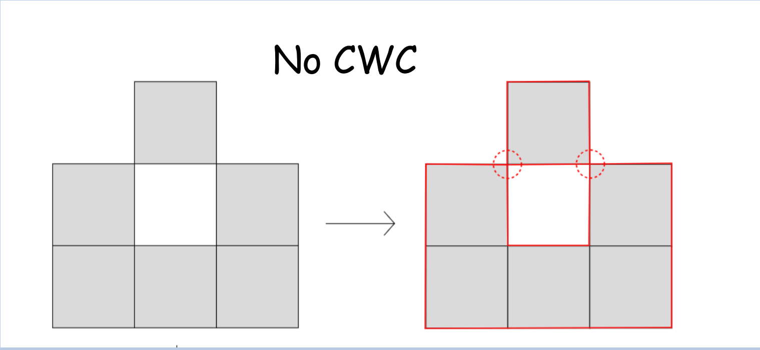
Starting point:



nD picture I

Associated cubical complex Q(I)

An nD cell complex is continuously well-composed (**CWC**) if the boundary of its continuous analog is an (n-1)-manifold.



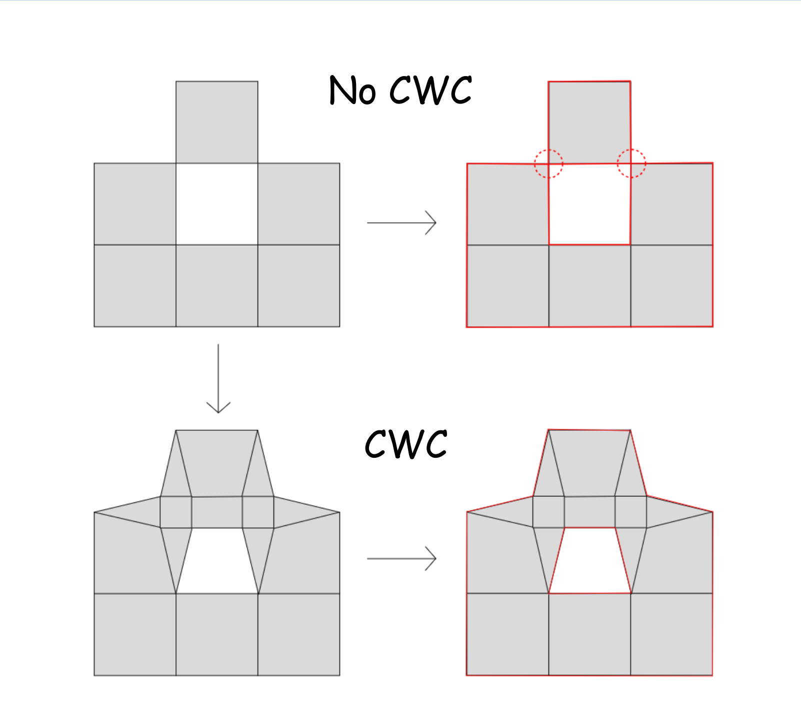
MOTIVATION:

CWC representation of an object enjoys some advantages:

- connected components of the boundary are Jordan (n-1)-D « surfaces » => they separate the ambient space into an interior (bounded) and an exterior (unbounded)

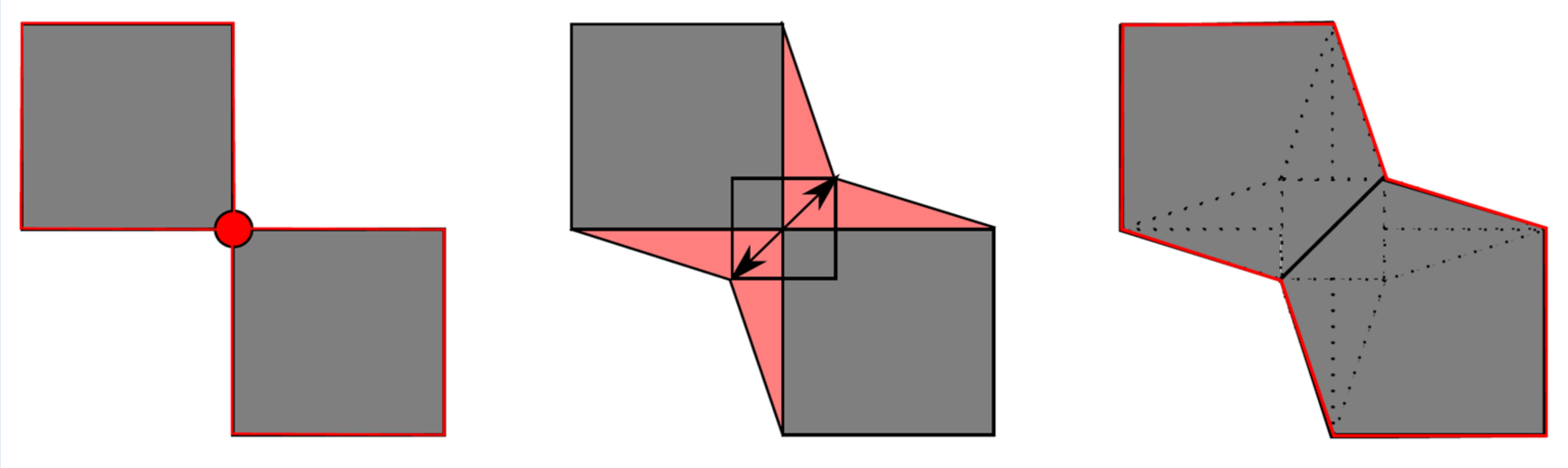
- Topological and geometrical computation benefits

An nD cell complex is continuously well-composed (**CWC**) if the boundary of its continuous analog is an (n-1)-manifold.



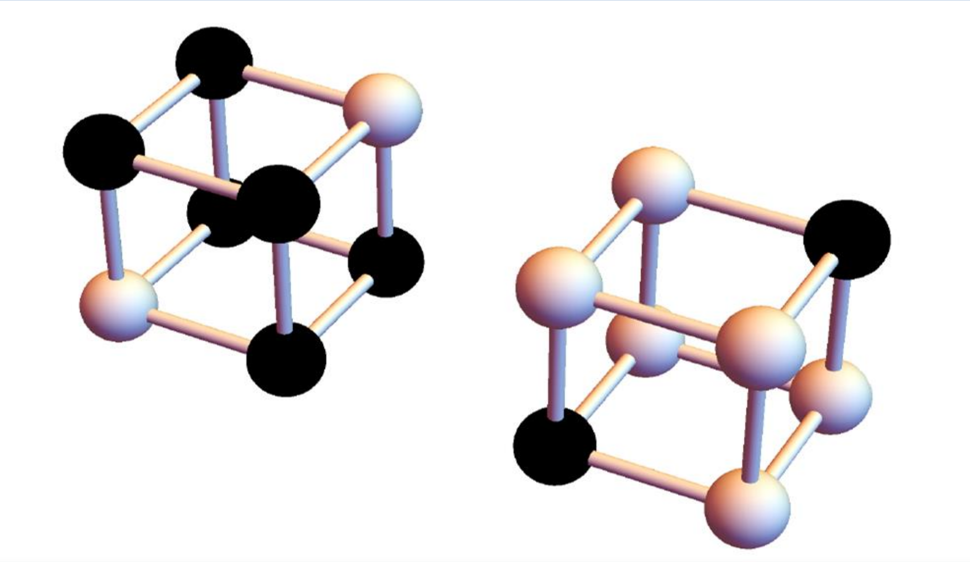
Homotopy equivalent

No CWC → « thickening » → CWC



The 2D/3D repairing method of Gonzalez Diaz *et al. 2015*

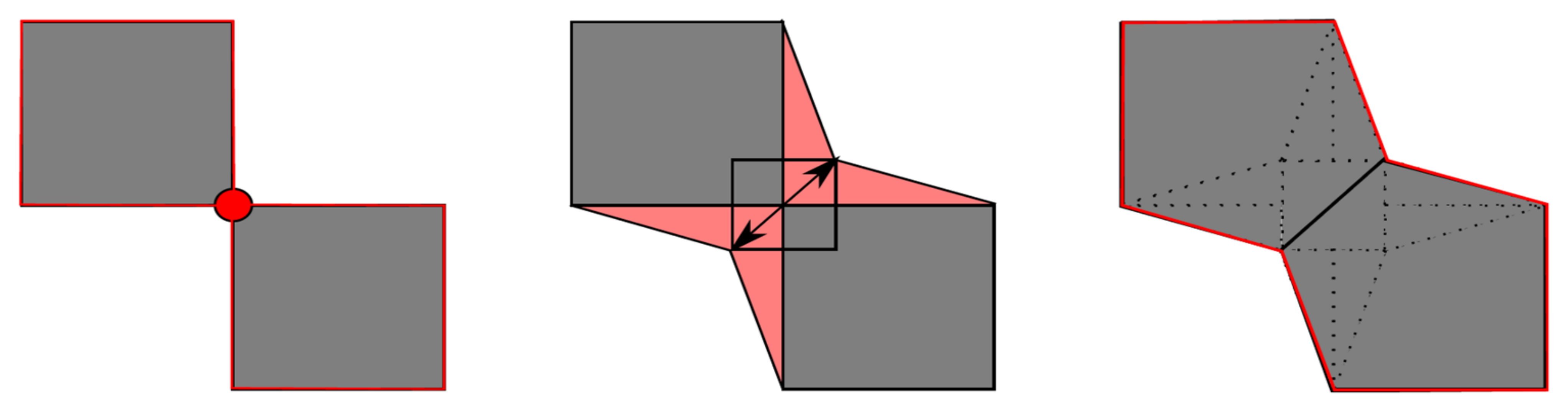
No CWC ⟺ No DWC ⟺ critical configurations



The 2D/3D repairing method of Gonzalez Diaz *et al. 2015*

No CWC ⟺ no DWC ⟺ critical configurations

Combinatorial method: find and repair critical configurations

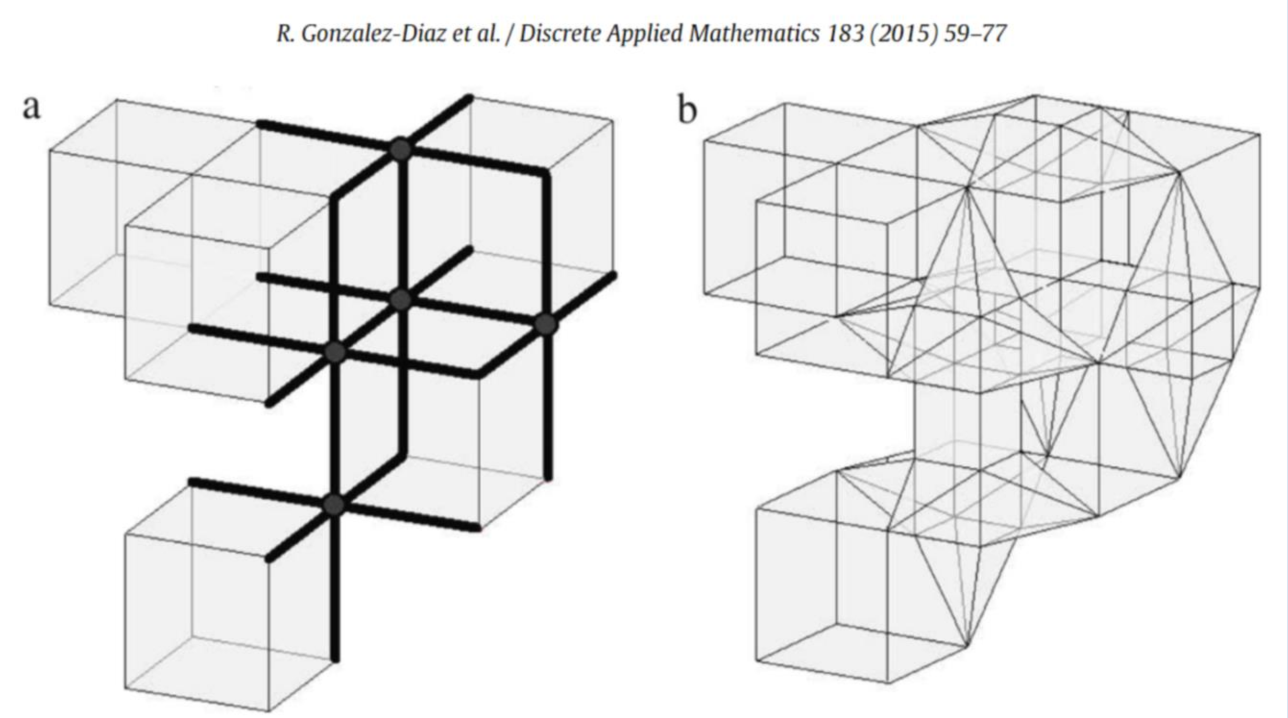


The 2D/3D repairing method of Gonzalez Diaz *et al.*

No CWC ⟺ no DWC ⟺ critical configurations

Combinatorial method: find and repair critical configurations

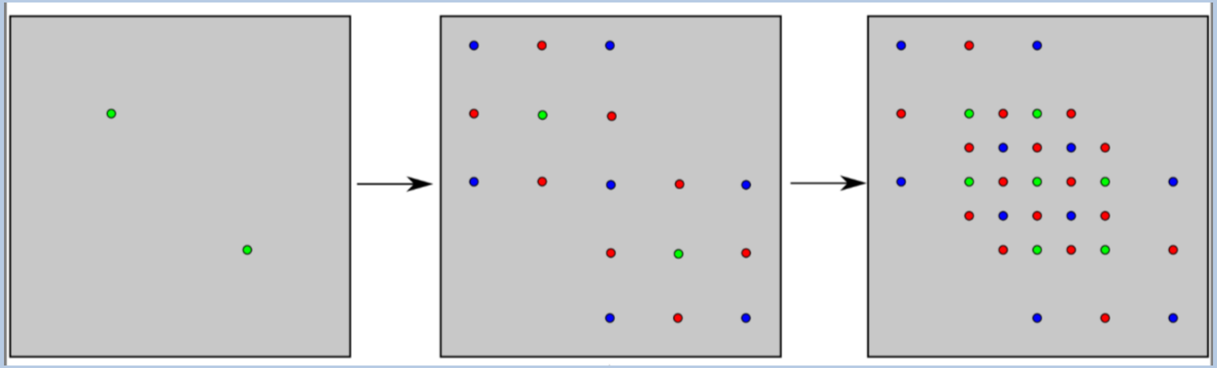
No CWC
3D Cubical
complex



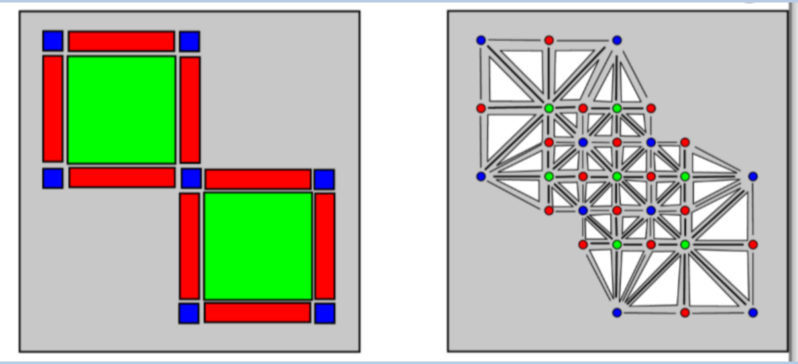
CWC
3D Cell
complex

Replicate the method for the **nD case**

nD picture I

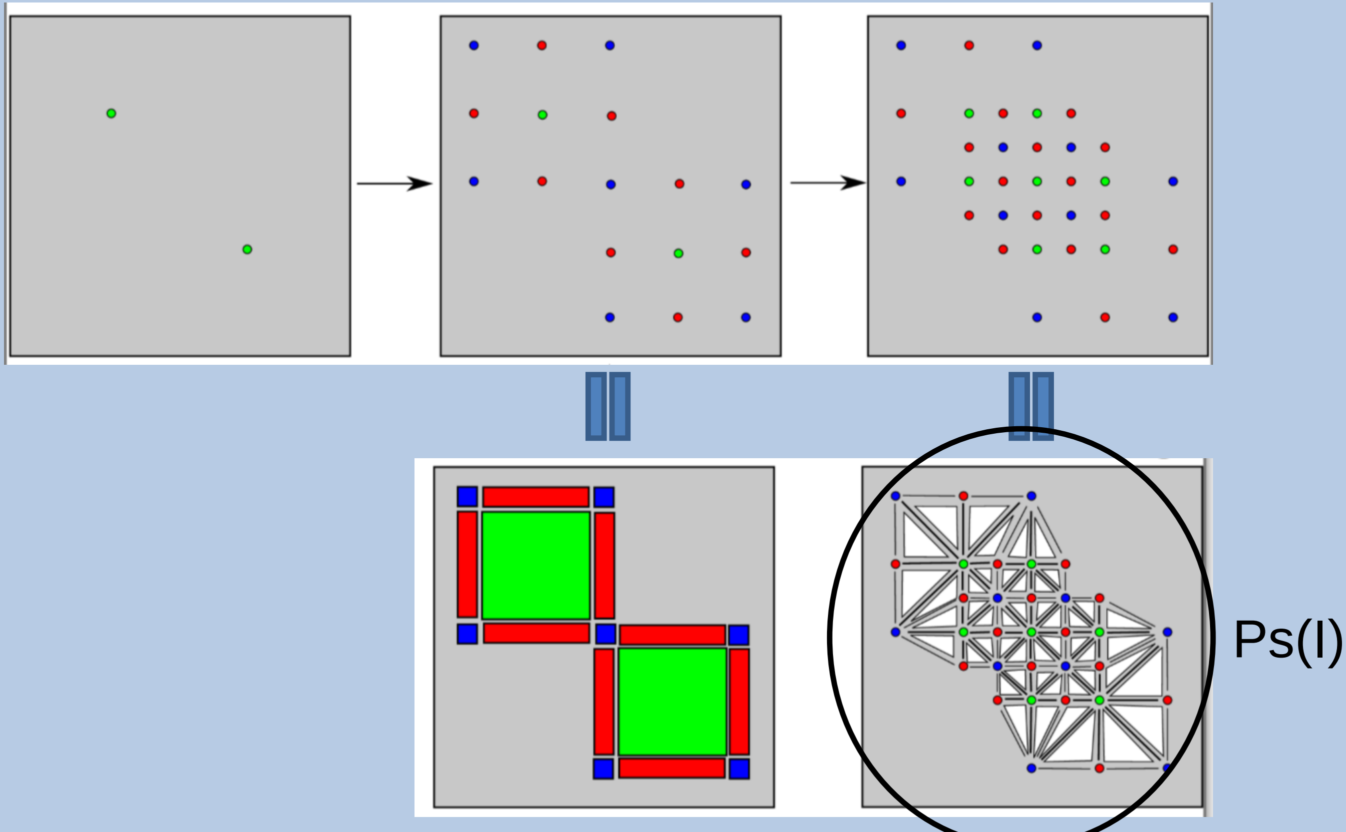


nD cubical
complex Q(I)

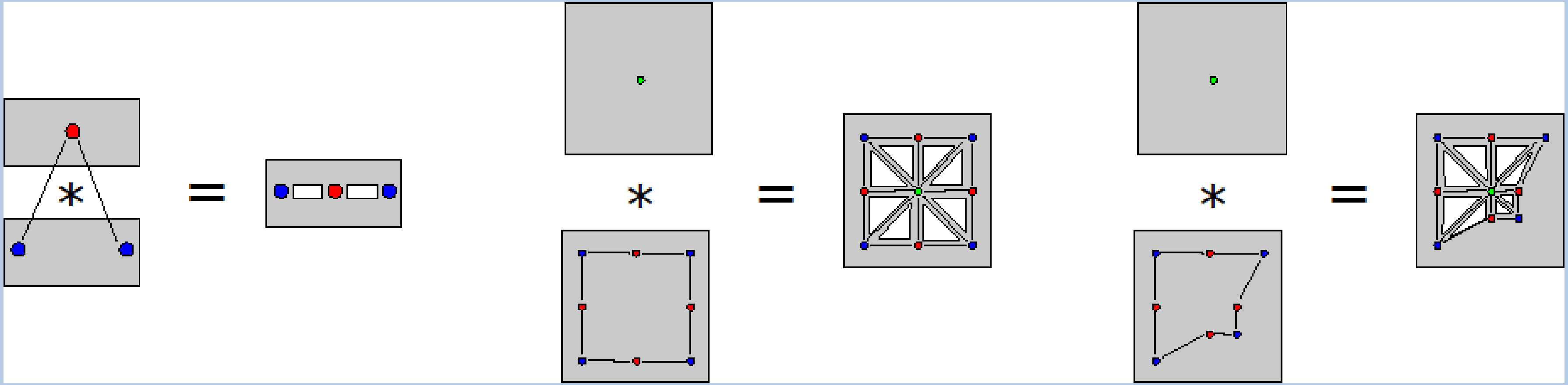


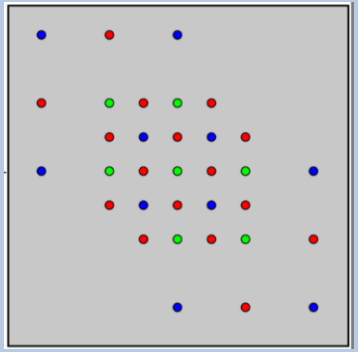
nD simplicial
complex Ps(I)
homotopy
equivalent to Q(I)

Replicate the method for the **nD case**

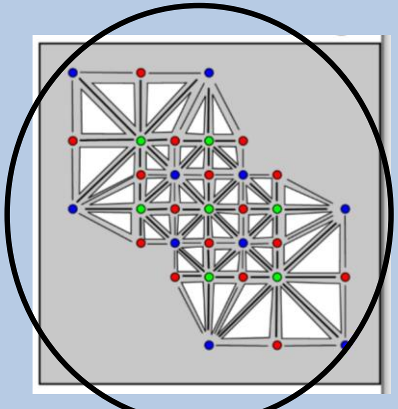


Replicate the method for the **nD case**





Cone joins



Ps(I)

BUT

Repairing critical configurations in nD does not guarantee CWCness

Conjecture: CWC $\Longrightarrow$ DWC

DWC $\not\Longrightarrow$ CWC

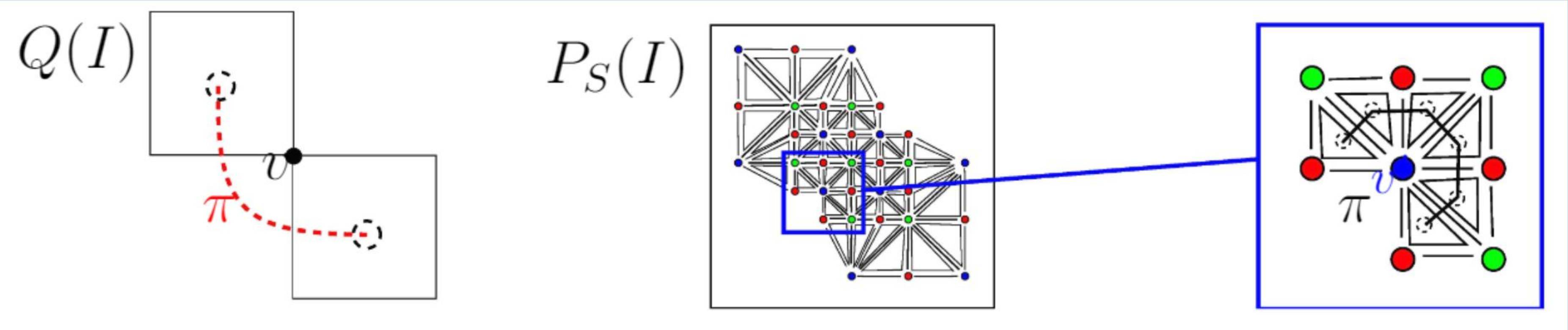
Much more difficult!!!

∂Ps(I) is an (n-1)-manifold??? Hard!

∂Ps(I) is a combinatorial (n-1)-manifold??? Hard!

∂Ps(I) is wWC??? Done!

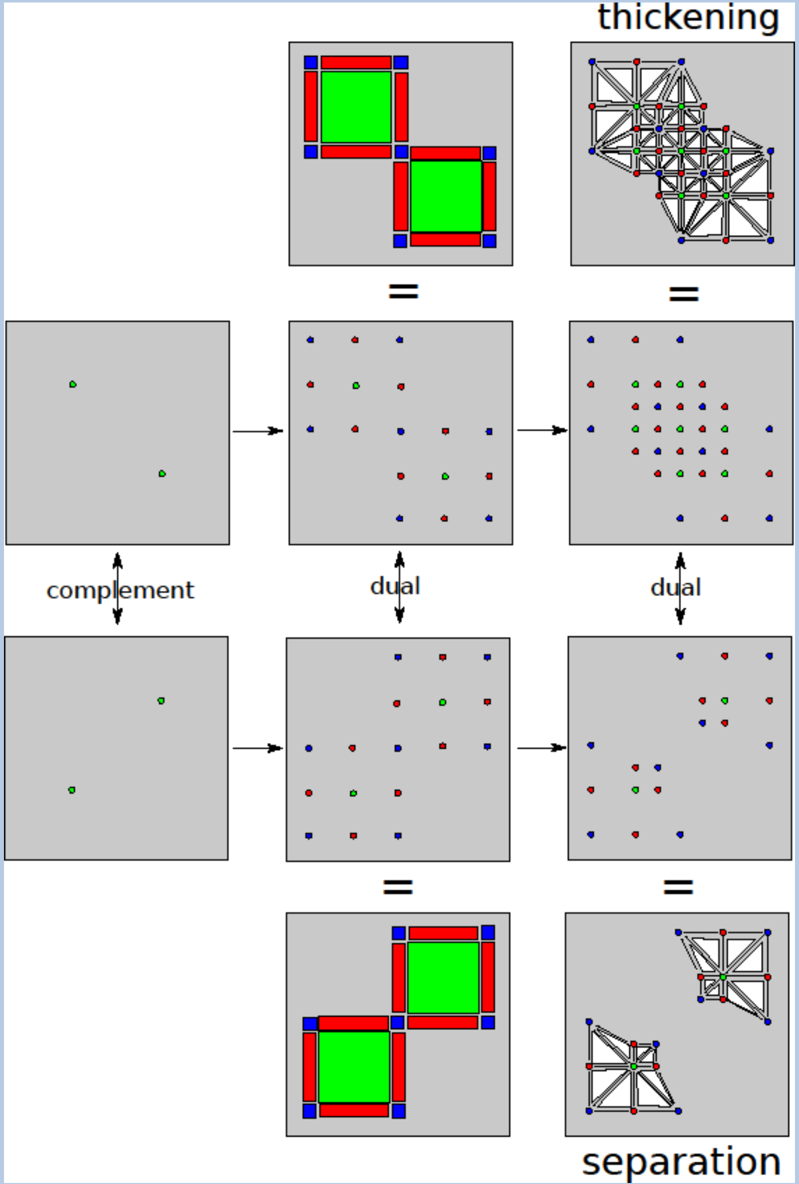
wWC = weakly Well-Composed



No wWC

wWC

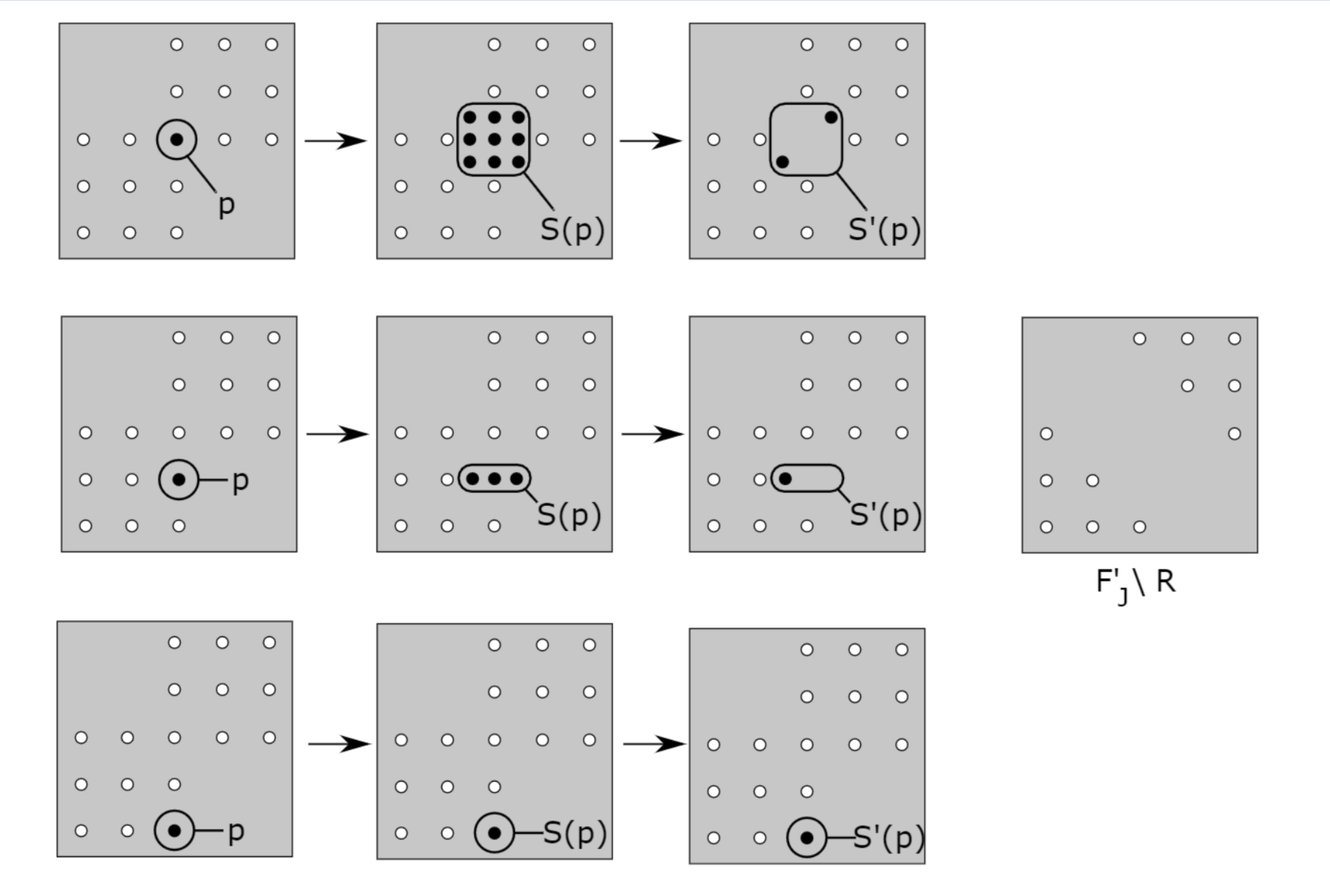
Path of face-connected n-simplices



Ps(I)

Ps($\bar{I}$)

Computation of Ps(I)
vs
computation of Ps($\bar{I}$)



wWC is self-dual:

$Ps(I)$ wWC

$Ps(\bar{I})$ wWC

$|Ps(\bar{I})| \cup |Ps(I)| = R^n$

$Ps(\bar{I}) \cap Ps(I) = \partial Ps(\bar{I}) = \partial Ps(I)$

Future work:

- to study the combinatorial structure of $\partial Ps(I)$
- to prove? that it is a combinatorial (n-1)-manifold

THANKS!!!