A spatial convexity descriptor for object enlacement

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Goal

A spatial convexity descriptor is designed and extended to complex spatial relations between objects like enlacement and interlacement.

Q-Convexity

The four quadrants around a point M in the integer lattice grid $\mathcal{G}$ are

$$Z_0(M) = \{N \in \mathcal{G} : \ 0 \le x_N \le x_M, \ 0 \le y_N \le y_M\} ,$$
$$Z_1(M) = \{N \in \mathcal{G} : \ x_M \le x_N < m, \ 0 \le y_N \le y_M\} ,$$
$$Z_2(M) = \{N \in \mathcal{G} : \ x_M \le x_N < m, \ y_M \le y_N < n\}$$
$$Z_3(M) = \{N \in \mathcal{G} : \ 0 \le x_N \le x_M, \ y_M \le y_N < n\} .$$

Let us denote the number of object points (foreground pixels) of F in $Z_p(i,j)$ by $n_p(i,j)$, for $p = 0, \ldots, 3$, i.e.,

$$n_p(i,j) = card(Z_p(i,j) \cap F\}) \ (p = 0, \ldots, 3) \ . \qquad (1)$$

Definition. *A lattice set F is Quadrant-convex (shortly, Q-convex) if for each (i,j) $(n_0(i,j) > 0 \wedge n_1(i,j) > 0 \wedge n_2(i,j) > 0 \wedge n_3(i,j) > 0)$ implies $(i,j) \in F$.*

A quantitative Q-concavity descriptor

If F is not Q-convex, then there exists a position (i,j) violating the Q-convexity property, i.e. $n_p(i,j) > 0$ for all $p = 0, \ldots, 3$ and $(i,j) \notin F$. We define the Q-concavity measure of F as the sum of the contributions of non-Q-convexity for each point in $\mathcal{R}$. Formally,

$$\varphi_F(i,j) = n_0(i,j)n_1(i,j)n_2(i,j)n_3(i,j)(1 - f(i,j)) \ , \qquad (2)$$

where (i,j) is an arbitrary point of $\mathcal{R}$, and $f(i,j) = 1$ if the point in position (i,j) belongs to the object, otherwise $f(i,j) = 0$. Moreover,

$$\varphi_F = \varphi_F(F) = \sum_{(i,j) \in \mathcal{R}} \varphi_F(i,j) \ . \qquad (3)$$

In order to measure the degree of Q-concavity, we normalize φ so that it ranges in $[0,1]$. We propose two possible normalizations (left global, right local) gained by normalizing each contribution.

$$\mathcal{E}_F^{(1)}(i,j) = \frac{\varphi_F(i,j)}{\max_{(i',j') \in \mathcal{R}} \varphi_F(i',j')} \qquad \mathcal{E}_F^{(2)}(i,j) = \frac{\varphi_F(i,j)}{((card(F) + h_i^F + v_j^F)/4)^4}$$

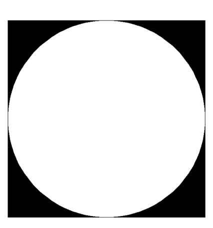
Then we sum up each single contribution and we divide by the number of non-zero contributions.

Definition. *For a given binary image F,*

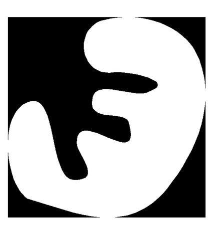
$$\mathcal{E}_F^{(\cdot)} = \sum_{(i,j) \in \bar{F}} \frac{\mathcal{E}_F^{(\cdot)}(i,j)}{card(\bar{F})}$$

Remark: The intersections of F with the four quadrants Z_0, Z_1, Z_2, Z_3 are an extension of the concept of *longitudinal cut* to two dimensions, and so relation (2) gives a quantification of the *enlacement* by the reference object F for the *landscape* point (i,j).

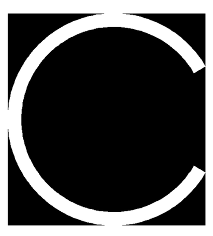
where $\bar{F}$ denotes the subset of (landscape) points in $\mathcal{R} \setminus F$ for which the contribution is not null.



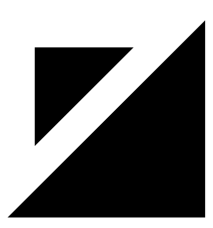
$\mathcal{E}_F^{(1)} = 0$
$\mathcal{E}_F^{(2)} = 0$



$\mathcal{E}_F^{(1)} = 0.38225$
$\mathcal{E}_F^{(2)} = 0.31707$



$\mathcal{E}_F^{(1)} = 0.56903$
$\mathcal{E}_F^{(2)} = 0.56773$



$\mathcal{E}_F^{(1)} = 0.80363$
$\mathcal{E}_F^{(2)} = 0.65960$

Object enlacement and interlacement

Let F and G be two objects. How much is G enlaced by F? The idea is to capture how many occurrences of points of G are somehow between points of F. Therefore, $\varphi_{FG}(i,j) = \varphi_F(i,j)$ if $(i,j) \in G$, and 0 otherwise. The enlacement descriptors of G by F are thus

$$\mathcal{E}_{FG}^{(1)}(i,j) = \frac{\varphi_{FG}(i,j)}{\max_{(i,j) \in G} \varphi_{FG}(i,j)} \qquad \mathcal{E}_{FG}^{(2)}(i,j) = \frac{\varphi_{FG}(i,j)}{((card(F) + h_i^F + v_j^F)/4)^4}$$

We may combine the enlacement of two objects by their harmonic mean to give a description of mutual enlacement (interlacement): $\mathcal{I}_{FG}^{(\cdot)} = \frac{2\mathcal{E}_{FG}^{(\cdot)}\mathcal{E}_{GF}^{(\cdot)}}{\mathcal{E}_{FG}^{(\cdot)} + \mathcal{E}_{GF}^{(\cdot)}}$

$\mathcal{E}_{FG}^{(2)} = 0$, $\mathcal{E}_{GF}^{(2)} = 0.93332$, $\mathcal{I}_{FG}^{(2)} = 0$

$\mathcal{E}_{FG}^{(2)} = 0.52357$, $\mathcal{E}_{GF}^{(2)} = 0.52856$, $\mathcal{I}_{FG}^{(2)} = 0.52605$

Experiments

In the experiments the two different interlacement values are computed such that F is the foreground and G is the background.

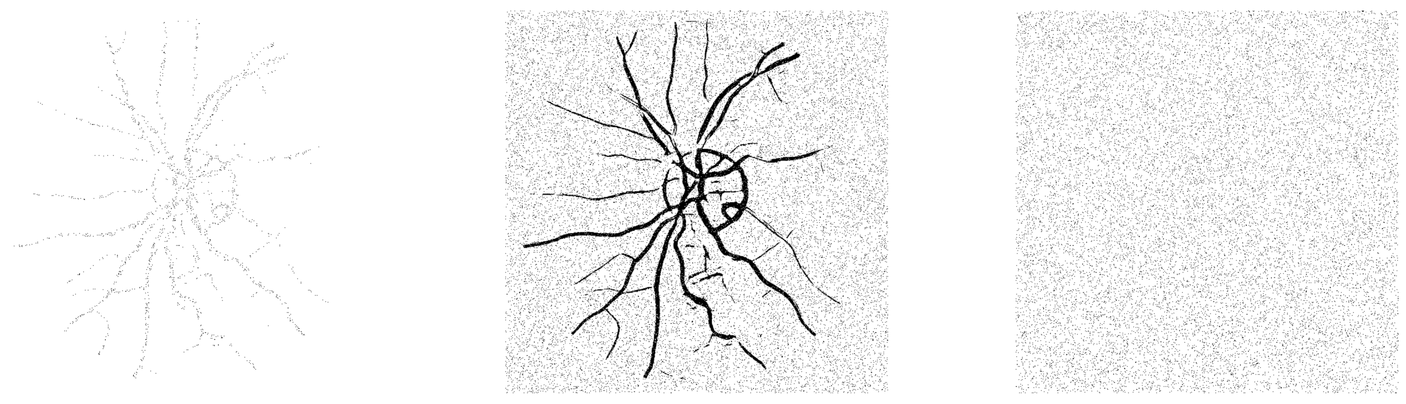
Scale tolerance Test

- We digitized 14 images on different scales $(32 \times 32, 64 \times 64, 128 \times 128, 256 \times 256, 512 \times 512)$
- We computed the average of the measured interlacement differences over the 14 pairs of consecutive images.
- Since the differences are of order 10^{-3}, we deduce that scaling has no significant impact on these measures, in practice, although in lower resolutions small parts of the shapes may disappear, and the differences can be higher.

Classification Task 1

- Databases: CHASE (20 binary images with centerd optic disk) and DRIVE (20 binary with shifted optic disk)
- We gradually added different types of random noise (Speckle, salt & pepper, and Gaussian noise, in the figure from left to right, respectively) to the images of size 1000×1000:

→ Gaussian and Speckle noise were added with 10 increasing variances $\sigma^2 \in [0, 2]$.
→ Salt & pepper noise was added with 10 increasing amounts in $[0, 0.1]$.

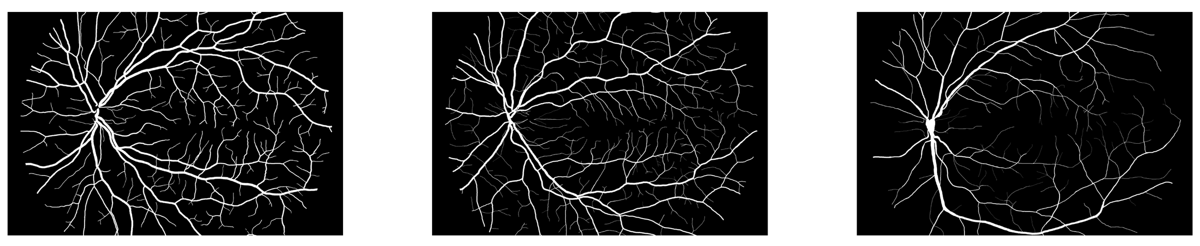


Then, we tried to classify the images into two classes (CHASEDB1 and DRIVE) based on their interlacement values, by the 5 nearest neighbor classifier with inverse Euclidean distance (5NN). We used leave-one-out cross validation to evaluate accuracy reported in the table:

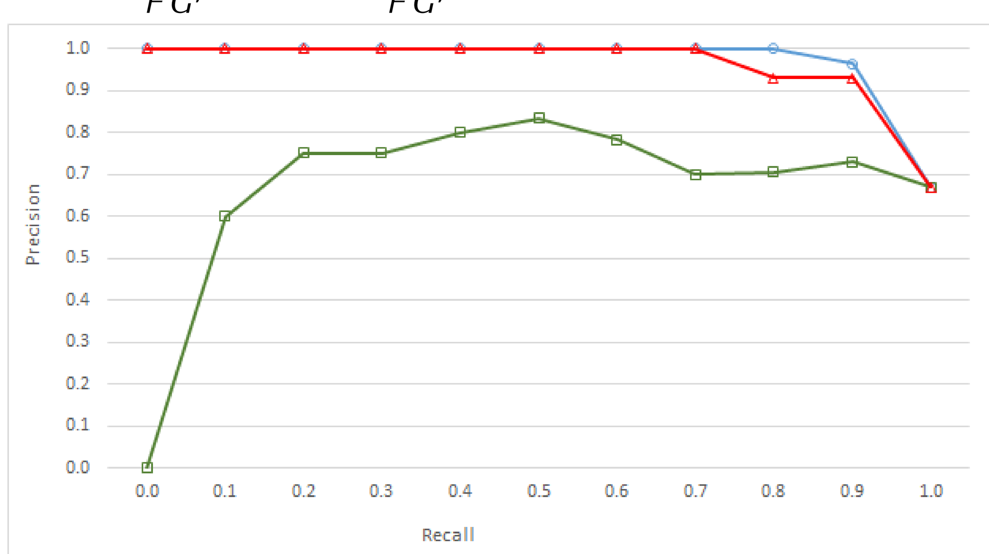
	Speckle		Salt & pepper		Gaussian	
	$\mathcal{I}_{FG}^{(1)}$	$\mathcal{I}_{FG}^{(2)}$	$\mathcal{I}_{FG}^{(1)}$	$\mathcal{I}_{FG}^{(2)}$	$\mathcal{I}_{FG}^{(1)}$	$\mathcal{I}_{FG}^{(2)}$
Level 1	60.0	95.0	87.5	92.5	65.0	95.0
Level 2	85.0	95.0	85.0	95.0	72.5	87.5
Level 3	60.0	95.0	85.0	95.0	95.0	80.0
Level 4	67.5	95.0	87.5	95.0	82.5	67.5
Level 5	82.5	95.0	85.0	95.0	67.5	47.5
Level 6	70.0	95.0	87.5	95.0	67.5	52.5
Level 7	47.5	95.0	85.0	92.5	52.5	50.0
Level 8	57.5	95.0	85.0	90.0	72.5	57.5
Level 9	55.0	95.0	85.0	90.0	32.5	55.0
Level 10	67.5	95.0	87.5	90.0	57.5	55.0

Classification Task 2

- Dataset: High-Resolution Fundus (HRF) composed of 45 images of fundus: 15 healthy, 15 with glaucoma symptoms and 15 with diabetic retinopathy symptoms.
- Using the same classifier as before we tried to separate the 15 healthy images from the 30 diseased cases.



Here we show the precision-recall curves obtained for this classification problem.
In Green: $\mathcal{I}_{FG}^{(1)}$, in Blue: $\mathcal{I}_{FG}^{(2)}$, and in Red: curve of Clement et al..



Summary

- A quantitative Q-concavity descriptor for complex spatial relations like enlacement and interlacement
- A fully two-dimensional approach: just using two directions (so four quadrants), we reached comparable and even better results than other methods employing many directions
- Various experiments illustrate the properties of the measure
- Future development: Preprocess the image by computing the principal axes and rotate the image to align principal axes and coordinate axes

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